Time to perform the physical observations of asteroids

Jian Gao (gaojian@bac.pku.edu.cn)

Department of Astronomy, Beijing Normal University, Beijing 100875, P. R.
CHINA

Jin Zhu (jinzhuobao.ac.cn)

National Astronomical Observatories, CAS, Beijing 100012, P. R. CHINA

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Abstract. The astrometric observations of asteroids are often performed near their opposition, where they have faster apparent motions and brighter visual magnitudes. However, the physical observations of asteroids (photometric and spectral) often require longer exposure times as well as brighter magnitudes for better signal-to-noise ratio, which are combined effects of both apparent moving speeds and visual magnitudes. We derive the equations of the apparent motion of asteroid in geocentric ecliptic coordinates. Comparison of the apparent magnitudes of asteroid at opposition and at stationary shows that the magnitude differences in the two cases are around 1 magnitude in average for most main belt asteroids, but are much larger for Near Earth Asteroids. Combining with comparison of asteroid apparent motion, the proper time for asteroid physical observation in different cases are discussed.

Keywords: asteroid, apparent motion, asteroid physical observation
Abbreviations: CAS – Chinese Academy of Sciences; S/N – signal-to-noise ratio; NAOC – National Astronomical Observatories; CAS; BFOSC – Beijing Faint Object Spectrograph and Camera;

1. Introduction

The best time for asteroid astrometric observations is near opposition when the asteroids become brighter sharply (Bel'skaya and Shevchenko, 2000), and their apparent motions reach the maximum. The brighter apparent magnitude and faster apparent motion near opposition means that asteroids could be detected more efficiently, and their fast changing positions make the opposition a best time in general to perform astrometric observations for orbit improvement. Therefore, many asteroid searching programs with small sky coverage search for and follow up asteroids near opposition.

However, the time near opposition may not be suited for the physical observations of asteroids. Although bright magnitude at opposition is good for the physical observations, the apparent motion speed of asteroids might be too fast that it would influence the observations which require high S/N ratio. For this reason, we consider the time
near stationary because the minimal apparent motion rates happen when asteroids are near these positions. If the apparent magnitude difference between opposition and stationary is small for the target asteroid, the position near stationary could be more ideal for the physical observations. In order to study all the above influence, we derive the equations describing the apparent motion of asteroid travelling on an elliptical orbit in geocentric ecliptic coordinates. By using these equations, differences of the apparent motion and the apparent magnitude between the opposition point and the stationary point are calculated in order to discuss the best time for asteroid physical observations.

2. Equations of Apparent Motions

Most of the asteroids are located in the main belt between Mars and Jupiter and their apparent motions are similar to the outer planets. The apparent motions of Near-Earth Asteroids (NEAs) are different because their orbits would be close to earth’s orbit and some even cross the earth’s orbit. The Spacewatch asteroid searching program in USA has been successfully discovering NEAs based upon the differences in the elliptic rates of motion of these asteroids compared to the distant
Main Belt, Trojan and Centaur cousins (Jedicke, 1996; Gehrels and Jedicke, 1996).

The equations describing ecliptic longitudinal and latitudinal motions of asteroids were derived by some researchers (Bowell \textit{et al.}, 1990; Jedicke, 1996; Sykes and Moynihan, 1996). Some of these equations were derived under assumption and are essentially restricted to the opposition point (Bowell \textit{et al.}, 1990). The ones derived by Sykes and Moynihan (1996, pp. 399-406) were more general and they used orbital elements as the variables. Here equations using rectangular coordinate components are derived. These equations are explicit and complete expressions for the apparent motions in ecliptic longitude and latitude of any asteroid in an elliptical orbit viewed from the Earth, whether near or far from opposition. The characteristic of these equations is that we can directly use the precise terrestrial rectangular coordinates from the JPL DE405 ephemeris and get more precise results.

The equation variables are defined as follows,

\( \vec{r}_\oplus, \vec{r}_a \) : Heliocentric position vectors of the Earth and asteroid;

\( x_\oplus, y_\oplus, z_\oplus \) : Earth’s heliocentric coordinates;

\( \dot{x}_\oplus, \dot{y}_\oplus, \dot{z}_\oplus \) : the time rates of change of Earth’s heliocentric coordinates;

\( \dot{x}_a, \dot{y}_a, \dot{z}_a \) : asteroid’s heliocentric coordinates;

\( \ddot{x}_a, \ddot{y}_a, \ddot{z}_a \) : the time rates of change of asteroid heliocentric coordinates;
\[ x' = x_a - x_\oplus, \quad y' = y_a - y_\oplus, \quad z' = z_a - z_\oplus: \text{asteroid's geocentric coordinates;} \]
\[ \dot{x}', \dot{y}', \dot{z}': \text{the time rates of change of asteroid's geocentric coordinates;} \]
\[ \lambda_e, \beta_e: \text{geometric ecliptic longitude and latitude;} \]
\[ \dot{\lambda}_e, \dot{\beta}_e: \text{the time rates of change of geocentric ecliptic longitude and latitude;} \]
\[ \rho': \text{Earth-asteroid distance;} \]
\[ \dot{\rho}' : \text{the time rates of change of Earth-asteroid distance;} \]

The equations are as follows:

\[ \begin{aligned}
\lambda_e &= \tan^{-1}\left(\frac{y'}{x'}\right) \\
\beta_e &= \sin^{-1}\left(\frac{z'}{\rho'}\right)
\end{aligned} \tag{1} \]

\[ \begin{aligned}
\dot{\lambda}_e &= \left(\frac{\dot{x}'y' - y'\dot{x}'}{x'z'}\right) \cos^2 \lambda_e \\
\dot{\beta}_e &= \frac{\rho' \dot{z}' - z' \dot{\rho}'}{\rho'^2 \cos \beta_e}
\end{aligned} \tag{2} \]

The total ecliptic apparent motion rate viewed from the Earth is,

\[ u = \sqrt{\dot{\lambda}_e^2 \cos^2 \beta_e + \dot{\beta}_e^2} \]

\[ = \sqrt{\left(\frac{x' y' - y' x'}{x'^2 y'}\right)^2 \rho^4 \cos^4 \lambda_e \cos^4 \beta_e + \left(\frac{\rho' \dot{z}' - z' \dot{\rho}'}{\rho'^2 \cos \beta_e}\right)^2 \dot{\rho}^4 \cos^4 \beta_e} \tag{3} \]

When using the same opposition approximate conditions as what Bowell \textit{et al.} (1990, pp. 19-24) or Sykes and Moynihan (1996, pp. 399-406) have used, Equation (2) will reduce to the ones in Bowell \textit{et al.} (1990, pp. 19-24).
The stationary is the position where apparent motions of planets or asteroids change into retrograde motion from prograde motion or *vice versa*. At the stationary, the time rate of the change of asteroid’s ecliptic longitude is the minimum. Because of the inclination and the eccentricity of the asteroid orbit, the apparent motion in ecliptic latitude at stationary is not always minimum so that the total apparent motion rate at this time is usually not zero. Danjon (1980, pp. 197-207) described the equation to calculate the time of stationary under assumption that both the eccentricity and the inclination are zero. If we force the \( x'\dot{y}' - y'\dot{x}' = 0 \) in the Equation (2), then \( \dot{\lambda}_v = 0 \), the asteroid should be at stationary at this time. In addition, based upon Equation (2), it is not difficult to prove that the asteroid should be at stationary when the time rate of change of \( \lambda_v \) equal to zero.

When asteroid is at stationary, if we neglect the influence of the inclination and the eccentricity, the apparent motion of asteroid can approximately be

\[
\begin{align*}
\dot{\lambda}_v &= 0 \\
\dot{\beta}_v &= \frac{\dot{z}'}{\rho'}
\end{align*}
\]  

(4)
3. **Apparent Motion and Apparent Magnitude at Stationary and Opposition**

Using the Equation (2) and Equation (3), we calculate the apparent motion of asteroids as well as their apparent magnitudes to get the necessary data at one opposition and two stationaries during their retrograde motion periods. The calculation of apparent visual magnitudes is based on the two-parameter IAU magnitude system described by Bowell et al. (1989, pp. 524-556). The orbit elements are from the Asteroid FTP Service by Lowell Observatory (Bowell et al., 1994). The calculation was divided into two parts. One is all the 32,729 numbered asteroids before Jan. 2002; the other includes all the 1,646 discovered NEAs with derived orbits before Jan. 2002.

The statistic results for different cases are shown in Figure 1 and Figure 2. It can be seen that the apparent motions of most asteroids at opposition positions are 0.15 deg/day larger than their motions at stationary positions (Figure 1a), and at opposition the apparent magnitudes of most asteroids are almost all brighter than the ones at stationary for about 1 magnitude (Figure 1b). But the apparent motion differences for NEAs can be 1.2 deg/day or even larger, and mostly between 0 and 0.7 deg/day (Figure 2a). There are some NEAs
with apparent magnitude differences larger than 1 or smaller than -2 magnitudes (Figure 2b), the latter caused by the fact that their orbits are much closer to the earth’s orbit.

Considering the apparent motion rates of all numbered asteroids, more than 91.95% asteroids have apparent motion rates larger than 0.2 deg/day at opposition, and more than 66.67% asteroids have apparent motion rates smaller than 0.05 deg/day at stationary point. At stationary, the apparent motions in ecliptic longitude are almost zero, and the small apparent motion rates mostly come from the apparent motions in ecliptic latitude. Therefore it is easier to track asteroid near stationary. The advantages of the slower apparent motion at stationary for the physical observations will be discussed in the following section. The histograms of the ratio of apparent motions between opposition and stationary are shown in Figure 3.
Figure 1. Comparison of 32,729 asteroids’ apparent motions and apparent magnitudes between opposition and stationary. (a) Histogram of the difference of apparent motion. (b) Histogram of the difference of apparent magnitude.

Figure 2. Comparison of 1,646 NEAs’ apparent motions and apparent magnitudes between opposition and stationary. (a) Histogram of the difference of apparent motion. (b) Histogram of the difference of apparent magnitude.
4. Best Time for Physical Observations of Asteroids

Two main goals of astrometric observation of asteroids are discovery and follow-up observations for orbital improvement, so the opposition position where asteroid has in general brighter apparent magnitude and faster apparent motion could be more efficiency. However physical observations of asteroids emphasize more on the signal-to-noise ratio (S/N) of the detection, requiring slow apparent motions as well as bright apparent magnitudes. The S/N of an object detected on a CCD image could be written as (Liu and Huang, 1996):
\[
S/N = \frac{N_{st}}{\sqrt{(N_{st} + N_{sky}) + (D^2 + R^2)^{1/2}}}
\]

where \(N_{st}\) is the signal count of target object, \(N_{sky}\) is the count of sky background, D is dark current during the exposure time and R is the bias.

Denoting the apparent magnitude of sky background with \(m_{sky}\), the apparent magnitude of asteroid at stationary with \(m_s\), and the one at opposition with \(m_o\), then we can have \(N_{ast s} \propto 2.512^{-m_s} \cdot t_s\), \(N_{ast o} \propto 2.512^{-m_o} \cdot t_o\), \(N_{sky s} \propto 2.512^{-m_{sky}} \cdot t_s\), \(N_{sky o} \propto 2.512^{-m_{sky}} \cdot t_o\), where \(N_{ast s}\) is the count of the asteroid image on CCD at stationary, \(N_{ast o}\) is the count of the asteroid image at opposition and \(N_{sky s}\) and \(N_{sky o}\) are the counts of sky background at stationary and at opposition respectively.

Introducing a variable \(L\) being equal to the ratio of the \(S/N\) values between stationary and opposition, and ignoring \(D\) and \(R\) in Equation (5), we can get

\[
L = \frac{(S/N)_{STA}}{(S/N)_{OPP}} = \frac{2.512^{m_o - m_s} \cdot \sqrt{2.512^{-m_o} + 2.512^{-m_{sky}}}}{\sqrt{2.512^{-m_s} + 2.512^{-m_{sky}}}} \sqrt{\frac{u_o}{u_s}}
\]

where \(u_s\) and \(u_o\) are the total apparent motion rates at stationary and at opposition respectively and \(t_s\) and \(t_o\) correspond the time of the asteroid’s image staying in certain CCD pixels respectively.
When observing bright asteroids from dark sites \( (m_{sky} \gg m_s) \), the above equation could be reduced to

\[
L = \sqrt{2.512 m_\nu - m_\nu \frac{u_0}{u_s}}
\]  \hspace{1cm} (7)

Figure 4 gives the histograms of L based on Equation (6) for the 32,729 numbered asteroids and the 1,646 NEAs. It can be seen that for about 91.1% asteroids of the 32,729 numbered asteroids, when the sky background is the same, the S/N of observations at stationary are better than the ones at opposition (Figure 4a). So when the target asteroid is a main belt asteroid, it would be better for choosing the time of stationary to perform physical observations. However it can be seen from the Figure 4b that there are 50.8% of the 1,646 NEAs having the value of L less than 1. So stationary might not be the best position to perform physical observations for the case of NEAs.

5. Conclusions and Discussions

For the case of the spectral observation, the image of the observed asteroid must be kept inside the incidence slit of the spectrograph during the exposure. The spectral observation systems can have two
a. 

Figure 4. Histogram of the variable L. (a) Result of 32,729 numbered asteroids. (b) Result of all NEAs. Note that the calculation includes two stationary points and one opposition. And the calculation assumed the sky background is 21 magnitudes.

different guiding methods. One system is that the target asteroid can be directly used as the guiding star, as for the case of the Cassegrain spectrograph system on the 2.16-m telescope at Xinglong station of the National Astronomical Observatories (NAOC), Chinese Academy of Sciences (CAS). The influence from apparent motion could be eliminated by guiding (except for some very fast moving NEAs), so the observation could be better made when the asteroid is brighter, where corresponds to the opposition for the case of most asteroids.

Sometimes the target asteroid could not be used as the guiding object, while another nearby bright object should be chosen in such system, like the case of the BFOSC system on NAOC 2.16-m tele-

SUBMIT2.tex; 2/07/2002; 20:28; p.13
scope. It is important to have the asteroid keeping no relative motion to background stars during the exposure, so the stationary would be the better choice for such purpose.

The spectral observations of NEAs are quite specific. They may have fast apparent motions even when they are far away from their opposition positions, which increases the difficulty for guiding. However, because of their short observable opportunity, there may have few choice on their practical observational time.

The photometric observation of asteroid with traditional photo-electric multiplier system like the 3-channel or 4-channel system on the NAOC 0.85-m telescope, has quite similar situation as the second case of the spectral observation, i.e., they have to be performed near the stationary. For the more general case of wide-field CCD photometric observations, like the current 2048×2048 CCD system on the NAOC 0.6-m Schmidt telescope, where the comparison stars are chosen in the same field with the target asteroid, the variable L defined in previous section could be used to get the position for better S/N. From statistics results of previous section, it could be seen that stationary is the better position for photometric observations of most asteroids. Making photometric observations near stationary for main belt asteroids with wide-field CCD also has an obvious advantage that the same set of
comparison stars could be used during different nights. In case of the $58' \times 58'$ field of view of the 2048×2048 CCD system on the NAOC 0.6-m Schmidt telescope, many asteroids could stay within the same field for about three weeks. The case for NEAs' photometric observations is much complex and may only be dealt with for each individual object.

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References


Address for Offprints

Jian Gao
Department of Astronomy,
Beijing Normal University,
Beijing,
100875, P. R. CHINA

Tel: +86-10- 62207236, Fax: +86 10 62763031
E-mail: gaojian@bac.pku.edu.cn