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BLOCK ADJUSTMENT OF A GROUP OF OVERLAPPING CCD IMAGES

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ABSTRACT

Since the field of view of a CCD is usually too small to cover enough reference stars, the block adjustment (BA) of CCD overlapping images is proposed in order to extend the sky coverage of observations, mitigate the effect of the position biases of reference stars, and consequently improve the local reference frame (LRF) of the observation. Observational equations of BA are given in vectorial expressions for the sake of easy understanding and programming. Investigation of the extrapolation of linear and nonlinear models illustrates that the nonlinear terms in CCD models should be determined and eliminated before implementing BA. The BA method is tested by simulated data as well as CCD observations, which shows that the application of BA to the overlapping observations could effectively improve the LRF of the observation and provide homogeneous results.

Key words: astrometry — methods: data analysis — reference systems

1. INTRODUCTION

As a relative measuring technique, traditional photographic astrometry can obtain positions of objects only referred to the plate local reference frame (LRF), which is represented by a set of reference stars of a certain catalog, rather than referred to the global reference frame (GRF), which is constructed by all stars of the catalog. It is obvious that the quality of the reference catalog affects the results of the photographic astrometry directly. If the reference stars are ideal in (1) precision of positions, (2) density of distribution, and (3) coverage of sky, reasonable results could be obtained however complicated the plate model is. However, in reality, such an ideal catalog is usually unavailable.

CCDs have been applied extensively in astronomy since the 1980s and have gradually replaced photographic plates because of their higher quantum efficiency, higher linearity, and greater convenience. But CCDs have a critical shortcoming, a relatively small field of view (FOV). Because of the small FOV of individual CCD images, they often contain few or no reference stars with precisely determined coordinates and proper motions. One has to find some secondary reference stars with lower precision in position and proper motion. The LRF in such cases could be biased from the GRF of the catalog in the form of translation, rotation, and distortion, which will affect the results of the objects. Independent solutions of single CCD observations could include very complicated systematic behaviors, a combination of which can lack global consistency. These factors seriously restrict the application of CCDs to astronomical researches characterized by large sky coverage.

To overcome the shortcoming mentioned above, i.e., to improve the LRF of CCD, is very important for their application to astrometric studies characterized by large sky coverage. As early as 1929, Donner & Furuhjelm (1929) proposed the idea of a combined solution of neighboring plates in the Helsingfors zone of the Atlas du Ciel by considering the characteristics of overlapping plates. In 1960 Eichhorn (1960) first introduced rigorous block adjustment (BA) of overlapping plates into astrometry. Since then BA has acquired an extensive literature (Abad & Garcóa 1995; Benevides-Soares & Teixerira 1992; Bustos Fierro & Calderón 2000, 2002; de Vegt & Ebner 1972, 1974; de Vegt 1991; Eichhorn et al. 1967; Eichhorn & Gatewood 1967; Eichhorn & Russell 1976; Googe et al. 1970; Jefferys 1963, 1979, 1987; Lukac 1967; Owen 1994; Stock 1981, 1992; Stock & Cova 1983; von der Heide 1977a, 1977b; Zacharias 1988, 1992), almost exclusively in connection with photographic astrometry. A thorough review, as well as an extensive bibliography on this subject, was given by Eichhorn (1988).

Compared with single plate adjustment (SPA), BA can avoid the contradiction of a star having several positions from various plates at a specified epoch. More importantly, because of the increase in number and sky coverage of reference stars, the LRF corresponding to the observations would be improved, which implies that BA may be an effective way to overcome this CCD shortcoming. The main purpose of this paper is to demonstrate how to apply BA and explore its effects.

In § 2, the observation equations of BA are given in vector expressions; the prerequisites for BA are analyzed in § 3; the BA method is tested by simulated data, as well as CCD observations, in §§ 4 and 5.

2. VECTOR EXPRESSIONS OF THE OBSERVATION EQUATIONS OF BA

In order to understand the basic principle of BA easily and develop the computer program conveniently, we give the observation equations of BA in vectorial expressions, which are different from those given by other authors like Stock (1981).

2.1. *Equations of Common Stars of Overlapping CCD Frames*

Suppose there are two overlapping CCD observations obtained at the epoch of t_1 and t_2, respectively. The mean positions of a common star at the same reference epoch (t_0) in the two CCD frames should be exactly the same, that is,

$$\boldsymbol{\rho}_{m1}(t_0) = \boldsymbol{\rho}_{m2}(t_0). \tag{1}$$

The relation between the mean position ($\boldsymbol{\rho}_m$) at the reference epoch and the observed position ($\boldsymbol{\rho}_o$) can be expressed as

$$\mathbf{T}_0'(\boldsymbol{\rho}_{m1} - \Delta\boldsymbol{\rho}_{\text{pm1}}) = (\text{PN})_1 \mathbf{T}_1'(\boldsymbol{\rho}_{o1} - \Delta\boldsymbol{\rho}_1), \qquad (2)$$

$$\mathbf{T}_0'(\boldsymbol{\rho}_{m2} - \Delta\boldsymbol{\rho}_{\text{pm2}}) = (\text{PN})_2 \mathbf{T}_2'(\boldsymbol{\rho}_{o2} - \Delta\boldsymbol{\rho}_2), \qquad (3)$$

where $\mathbf{T}_0$ is the triad of the mean equatorial coordinate system (Murray 1983), $\mathbf{T}_1$ and $\mathbf{T}_2$ are the triads of the instantaneous true equatorial coordinate system, $\Delta\boldsymbol{\rho}_{\text{pm1}}$ and $\Delta\boldsymbol{\rho}_{\text{pm2}}$ are the corrections of proper motion, $\Delta\boldsymbol{\rho}_1$ and $\Delta\boldsymbol{\rho}_2$ contain the corrections for parallax, aberration, gravitational deflection, and atmospheric refraction, and $(\text{PN})_1$ and $(\text{PN})_2$ are the precession-nutation transform matrices to convert the instantaneous true position to the mean position of the reference epoch.

The observed position could be expressed in the celestial coordinate system and in the standard coordinate system of CCD as

$$\mathbf{T}_1'\boldsymbol{\rho}_{o1} = \begin{pmatrix} \cos\delta_1 \cos\alpha_1 \\ \cos\delta_1 \sin\alpha_1 \\ \sin\delta_1 \end{pmatrix} = \mathbf{W}_{s1}(\alpha_1^0, \delta_1^0) k_1 \begin{pmatrix} \xi_1 \\ \eta_1 \\ 1 \end{pmatrix}, \qquad (4)$$

$$\mathbf{T}_2'\boldsymbol{\rho}_{o2} = \begin{pmatrix} \cos\delta_2 \cos\alpha_2 \\ \cos\delta_2 \sin\alpha_2 \\ \sin\delta_2 \end{pmatrix} = \mathbf{W}_{s2}(\alpha_2^0, \delta_2^0) k_2 \begin{pmatrix} \xi_2 \\ \eta_2 \\ 1 \end{pmatrix}, \qquad (5)$$

where the coefficients k_1 and k_2 are given by

$$k_1 = (1 + \xi_1^2 + \eta_1^2)^{-1/2}, \qquad (6)$$

$$k_2 = (1 + \xi_2^2 + \eta_2^2)^{-1/2}, \qquad (7)$$

(α_1, δ_1) and (α_2, δ_2) are celestial coordinates corresponding to the observed directions of the two observations of a common star, and $\mathbf{W}_{s1}(\alpha_1^0, \delta_1^0)$ and $\mathbf{W}_{s2}(\alpha_2^0, \delta_2^0)$ are transform matrices converting the standard coordinates on the tangent plane to the equatorial coordinate system and expressed as

$$\mathbf{W}_{s1} = \begin{pmatrix} -\sin\alpha_1^0 & -\sin\delta_1^0 \cos\alpha_1^0 & \cos\delta_1^0 \cos\alpha_1^0 \\ \cos\alpha_1^0 & -\sin\delta_1^0 \sin\alpha_1^0 & \cos\delta_1^0 \sin\alpha_1^0 \\ 0 & \cos\delta_1^0 & \sin\delta_1^0 \end{pmatrix}, \qquad (8)$$

$$\mathbf{W}_{s2} = \begin{pmatrix} -\sin\alpha_2^0 & -\sin\delta_2^0 \cos\alpha_2^0 & \cos\delta_2^0 \cos\alpha_2^0 \\ \cos\alpha_2^0 & -\sin\delta_2^0 \sin\alpha_2^0 & \cos\delta_2^0 \sin\alpha_2^0 \\ 0 & \cos\delta_2^0 & \sin\delta_2^0 \end{pmatrix}, \qquad (9)$$

where (α_1^0, δ_1^0) and (α_2^0, δ_2^0) are the coordinates of the tangential points of the two CCD frames, and § 3.3 discusses how to get them. The standard coordinates of the common star are (ξ_1, η_1) and (ξ_2, η_2). Generally, the relationship between the standard and the measured coordinates can be expressed as

$$\left\{ \begin{array}{l} \xi_1 = H(x_1, y_1; a_1, a_2, \ldots, a_n) \\ \eta_1 = I(x_1, y_1; a_1, a_2, \ldots, a_n) \end{array} \right\}, \qquad (10)$$

$$\left\{ \begin{array}{l} \xi_2 = H(x_2, y_2; b_1, b_2, \ldots, b_n) \\ \eta_2 = I(x_2, y_2; b_1, b_2, \ldots, b_n) \end{array} \right\}, \qquad (11)$$

where $a_1, a_2, \ldots, a_{\text{n}}$ and $b_1, b_2, \ldots, b_n$ are model parameters of the two CCD frames, and (x_1, y_1) and (x_2, y_2) are the measured coordinates of the common star.

Substituting equations (2) through (11) into equation (1) and considering the observation errors, we have

$$(\text{PN})_1 \left[\mathbf{W}_{s1} k_1 \begin{pmatrix} H(x_1, y_1; a_1, a_2, \ldots, a_n) \\ I(x_1, y_1; a_1, a_2, \ldots, a_n) \\ 1 \end{pmatrix} - \mathbf{T}_1'\Delta\boldsymbol{\rho}_1 \right] + \mathbf{T}_0'\Delta\boldsymbol{\rho}_{\text{pm1}}$$
$$= (\text{PN})_2 \left[\mathbf{W}_{s2} k_2 \begin{pmatrix} H(x_2, y_2; b_1, b_2, \ldots, b_n) \\ I(x_2, y_2; b_1, b_2, \ldots, b_n) \\ 1 \end{pmatrix} - \mathbf{T}_2'\Delta\boldsymbol{\rho}_2 \right] + \mathbf{T}_0'\Delta\boldsymbol{\rho}_{\text{pm2}} + v_i. \qquad (12)$$

Equation (12) is the constraint equation of the model parameters of two CCD frames provided by common stars for which the $\Delta\boldsymbol{\rho}_1$ and $\Delta\boldsymbol{\rho}_2$ can be obtained from SPA with some secondary reference stars in advance. If the CCD frames are taken successively, i.e., the time interval between two CCD overlapping frames is small (say, no more than 1 month), the effect of the proper motion could be negligible. But sometimes we have to reduce overlapping observations with time intervals of 1 yr or longer. In such cases it is recommended to refer to some secondary catalogs and only use the stars with known proper motions as common stars.

2.2. *Equations of Reference Stars*

For the reference stars within the sky area covered by the overlapping CCD frames, the observation equations could be expressed as

$$\left\{ \begin{array}{l} \xi(\alpha_0, \delta_0, \alpha, \delta) = H(x, y; a_1, a_2, \ldots, a_n) \\ \eta(\alpha_0, \delta_0, \alpha, \delta) = I(x, y; a_1, a_2, \ldots, a_n) \end{array} \right\}. \qquad (13)$$

The unknowns in the combined observation equations of the common stars and the reference stars are model parameters of all CCD frames. They can be solved with weighted least-square adjustment of the combined observation equations.

Based on the model parameters and the measured coordinates, the standard and celestial coordinates of objects can be calculated successively. For any object that appears on more than one CCD frame, the BA would finally give a unique position.

3. ANALYSIS OF THE PREREQUISITES OF BA WITH SIMULATED OBSERVATIONS

Equation (12) shows that the model parameters of a CCD could be expressed as the function of those of the overlapping CCD frames with the help of common stars. If the reference stars are not sufficient for a SPA, then the model parameters of the overlapping area (constrained by common stars) have to be taken as those of the whole CCD frame. For a CCD with a linear model, i.e., the relationship between the standard and the measured coordinates is linear, such extrapolation is reasonable. While for a CCD with a nonlinear model that describes the optical distortion (third order), the tilt of the CCD to focal plane (second order), the bias of the value of the adoption of the tangential point (second order), and so on, the model parameters fitted from a part of a CCD area have large errors and should not be applied by extrapolation to the whole CCD area (see § 3.2). Therefore, nonlinear terms in a

TABLE 1
Statistical Results of the Extrapolation of Linear and Nonlinear Models from Portion to the Whole of the CCD

			6 Parameter Model (mas)				12 Parameter Model (mas)				14 Parameter Model (mas)			
Coverage	$N_{\rm ref}$	Case[a]	$\overline{\lvert\Delta\alpha\rvert}$	$\sqrt{\overline{\sigma^2_{\Delta\alpha}}}$	$\overline{\lvert\Delta\delta\rvert}$	$\sqrt{\overline{\sigma^2_{\Delta\delta}}}$	$\overline{\lvert\Delta\alpha\rvert}$	$\sqrt{\overline{\sigma^2_{\Delta\alpha}}}$	$\overline{\lvert\Delta\delta\rvert}$	$\sqrt{\overline{\sigma^2_{\Delta\delta}}}$	$\overline{\lvert\Delta\alpha\rvert}$	$\sqrt{\overline{\sigma^2_{\Delta\alpha}}}$	$\overline{\lvert\Delta\delta\rvert}$	$\sqrt{\overline{\sigma^2_{\Delta\delta}}}$
1/4	30	1	10.8	42.8	15.5	46.0	76.5	147.3	90.1	172.6	230.7	351.4	138.7	281.3
		2	10.0	30.9	17.9	36.3	76.8	145.5	91.3	172.4	230.7	350.2	137.1	279.1
1/2	60	1	10.7	41.8	11.3	42.1	28.3	58.6	55.3	93.7	41.3	84.6	71.6	123.2
		2	11.0	29.9	11.2	28.2	26.5	51.5	55.8	90.7	39.8	79.6	72.1	119.1

Note.—Similar to Table 2 and Table 4.
[a] (1) Reduction results minus simulated catalog values (with random errors); (2) reduction results minus true values (without random errors).

CCD model should be determined and corrections applied before running a BA.

3.1. *Setup of Simulations*

In order to verify the analysis mentioned above, three types of CCD observations (FOV $= 1^\circ \times 1^\circ$, 2048×2048 pixels, scale $= 0.024$ mm pixel^{-1}) with linear and nonlinear models are simulated. First, 120 stars distributed uniformly in right ascension and declination are simulated within the FOV. Then, three series of measured coordinates are computed from the following three different models, respectively,

$$\left\{ \begin{array}{l} \xi = ax + by + c \\ \eta = a'x + b'y + c' \end{array} \right\}, \tag{14}$$

$$\left\{ \begin{array}{l} \xi = ax + by + c + px^2 + qxy + ry^2 \\ \eta = a'x + b'y + c' + p'x^2 + q'xy + r'y^2 \end{array} \right\}, \tag{15}$$

$$\left\{ \begin{array}{l} \xi = ax + by + c + px^2 + qxy + ry^2 + sx(x^2 + y^2) \\ \eta = a'x + b'y + c' + p'x^2 + q'xy + r'y^2 + s'y(x^2 + y^2) \end{array} \right\}, \tag{16}$$

where the values of the parameters, for example, are as follows: $a = 0.9664080$, $b = -0.0223537$, $c = 0.0$, $p = 0.0132360$, $q = -0.0130134$, $r = -0.0120570$, $s = 1.2$, $a' = 0.0223331$, $b' = 0.9664858$, $c' = 0.0$, $p' = 0.0110304$, $q' = 0.0119571$, $r' = -0.0121256$, and $s' = 2.4$.

Finally, random errors with a Gaussian distribution with standard deviation of $\sigma = 30$ mas are added to the star equatorial coordinates, and the resulting values are taken as the simulated catalog positions, while random errors with a standard deviation of $\sigma = 27$ mas are added to the measured coordinates.

3.2. *Investigation of the Extrapolation of Linear and Nonlinear Models*

First we investigate the situation of the extrapolation of the model from a portion to the whole of the CCD. We assume that the reference stars are distributed over one-fourth, or half, of the whole CCD. The reduction and catalog coordinates of objects are compared, and the mean and the rms of the differences are calculated by the following formulae:

$$\overline{\Delta\alpha \cos\delta} = \frac{\sum_{k=1}^{n} \Delta\alpha_k \cos\delta_k}{n}, \tag{17}$$

$$\sigma_{\Delta\alpha} = \sqrt{\frac{\sum_{k=1}^{n} \left(\Delta\alpha_k \cos\delta_k - \overline{\Delta\alpha \cos\delta}\right)^2}{n-1}}, \tag{18}$$

where $\Delta\alpha_k$ is the difference in the reduction and the catalog right ascension of the kth object, and n is the number of objects. To get statistically meaningful results, 20 simulations with different random numbers are performed. Table 1 lists the average of the absolute values of $\overline{\Delta\alpha \cos\delta}$ for 20 simulations, i.e., $\overline{\lvert\overline{\Delta\alpha}\rvert}$, and standard deviation of $\sigma_{\Delta\alpha}$ for 20 simulations, i.e., $(\overline{\sigma^2_{\Delta\alpha}})^{1/2}$. Those in declination are calculated similarly. The following line lists the corresponding results compared with the original equatorial coordinates without random errors. It is demonstrated from the table that in the case of the linear model the reduction results are in reasonable consistency with the error of the simulated data. However for the nonlinear model, even with the reference stars distributed within half of the CCD, the results are worse by a factor of 3 or more with respect to the linear case.

As to the extrapolation of the model from many CCD frames to one, various overlapping configurations of CCD frames are simulated as shown in Figure 1 in which Figure 1*a* is in "chain" form, Figure 1*b* is in "X" form, and Figure 1*c* is in "strip" form. The results obtained from BA of the objects on the central frame of each configuration are shown in Table 2, from which it is shown that, for the linear CCD model, although there are no reference stars on the central frame, the results of BA are still acceptable for three overlapping configurations. However, for the nonlinear model, the reduction results deviate from catalog coordinates (with or without random errors) remarkably compared with the noise level adopted in the simulated data. For the X configuration, the central CCD is overlapped and constrained completely by the surrounding four CCD frames; the BA results are better than those from the chain configuration for the nonlinear model but are still worse by a factor of 4 or more with respect to the linear case. In addition, because of the increase in the overlapping area, the BA results of the strip configuration for the nonlinear model are better than those from the chain configuration but again are still worse with respect to the linear case.

For different sets of values of model parameters, similar conclusions have been obtained. It can be summarized that an extrapolation (chain of overlapping CCD images with no reference stars) is feasible for the linear model, but unfeasible for the nonlinear model.

3.3. *Preparation of the Data for a BA*

Because the extrapolation of the nonlinear model contains unacceptable errors, preprocessing of the data should be done before BA to eliminate the influence of nonlinear terms.

In practice nonlinear terms such as field distortion and plate tilt are usually almost constant for a group of CCD

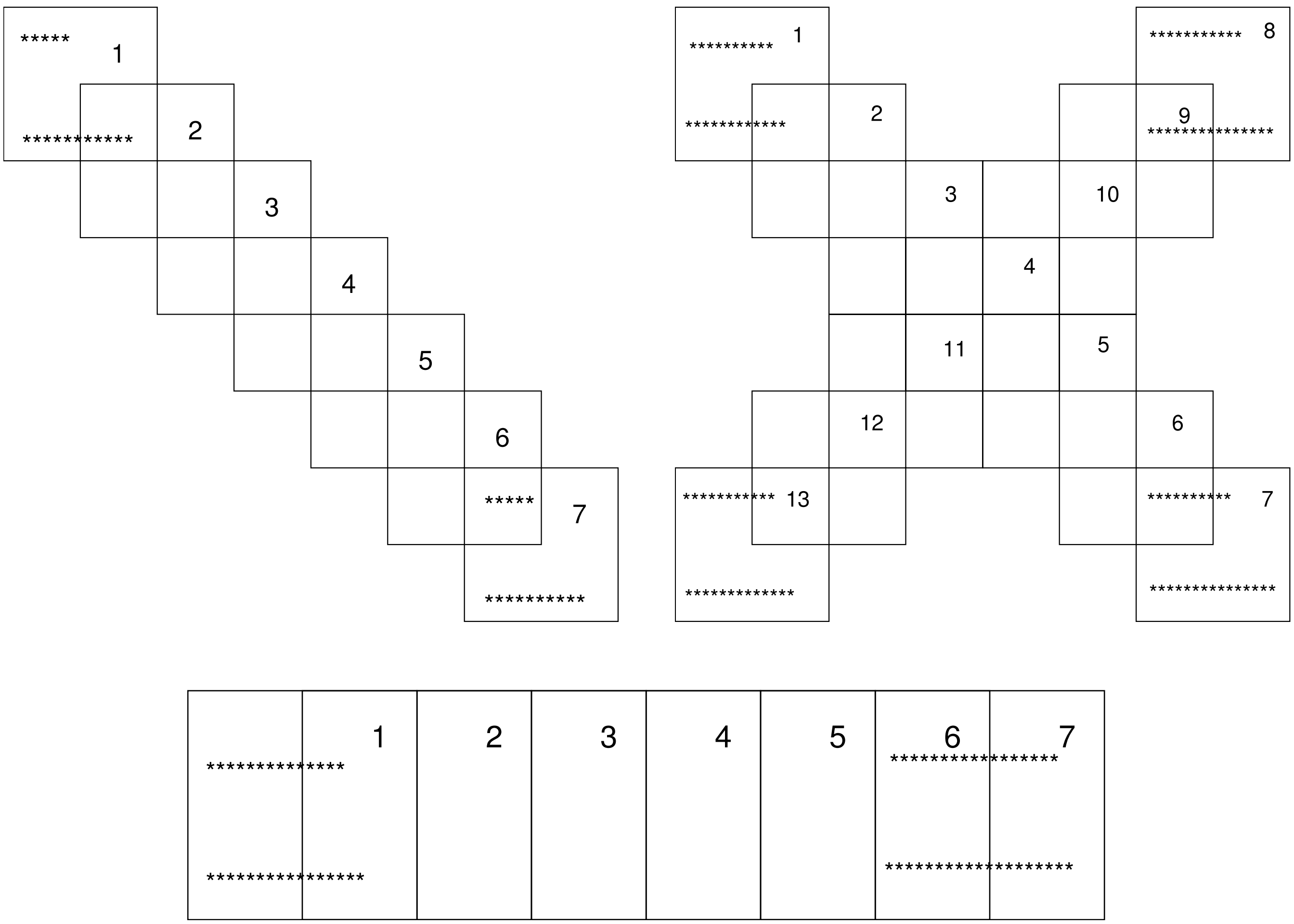

FIG. 1.—Configurations of simulated overlapping CCD observations. (*a*) "Chain" configuration of 7 CCD frames; (*b*) "X" configuration of 13 CCD frames; and (*c*) "strip" configuration of 7 CCD frames. The asterisks in the figures represent the reference stars.

TABLE 2

STATISTICAL RESULTS OF THE EXTRAPOLATION OF LINEAR AND NONLINEAR MODELS FROM MANY CCD FRAMES TO ONE FOR DIFFERENT CONFIGURATIONS SHOWN IN FIG. 1

No. CCD Frames (1)	Configuration (2)	No. Frames with Ref. Stars (3)	No. Ref. Stars (4)	6 Parameter Model (mas)				12 Parameter Model (mas)				14 Parameter Model (mas)			
				$\overline{\lvert\Delta\alpha\rvert}$	$\sqrt{\sigma^2_{\Delta\alpha}}$	$\overline{\lvert\Delta\delta\rvert}$	$\sqrt{\sigma^2_{\Delta\delta}}$	$\overline{\lvert\Delta\alpha\rvert}$	$\sqrt{\sigma^2_{\Delta\alpha}}$	$\overline{\lvert\Delta\delta\rvert}$	$\sqrt{\sigma^2_{\Delta\delta}}$	$\overline{\lvert\Delta\alpha\rvert}$	$\sqrt{\sigma^2_{\Delta\alpha}}$	$\overline{\lvert\Delta\delta\rvert}$	$\sqrt{\sigma^2_{\Delta\delta}}$
7..	Chain	1,7	240	25.9	39.6	23.7	48.3	153.3	108.8	121.0	125.1	352.1	207.1	173.7	174.0
				24.9	28.3	25.2	37.7	154.3	105.5	121.6	123.1	379.4	203.5	193.8	171.9
13......................................	X	8,13	480	17.1	37.8	11.3	41.3	108.0	53.2	96.7	61.1	177.7	91.4	294.8	119.9
		1,7		17.9	25.7	13.3	27.3	108.8	45.8	96.2	53.1	176.5	85.8	297.1	114.9
7..	Strip	1,7	240	17.0	40.8	21.8	41.0	55.6	44.4	68.1	51.3	98.3	53.4	73.6	50.2
				16.6	27.0	20.3	26.6	56.2	35.3	67.1	41.7	97.7	44.2	72.8	38.6

TABLE 3

FORM OF VARIOUS MODELS

$N_{\rm param}$	Models
4...	$\xi = Ax + By + C$
	$\eta = Ay - Bx + D$
6...	$\xi = Ax + By + C$
	$\eta = A'x + B'y + C'$
8...	$\xi = Ax + By + C + px^2 + qxy$
	$\eta = A'x + B'y + C' + pxy + qy^2$
12...	$\xi = Ax + By + C + px^2 + qxy + ry^2$
	$\eta = A'y + B'x + C' + p'x^2 + q'xy + r'y^2$
7...	$\xi = Ax + By + C + {\rm p}x^2 + qxy + rx(x^2 + y^2)$
	$\eta = Ay - Bx + D + pxy + qy^2 + ry(x^2 + y^2)$
9...	$\xi = Ax + By + C + px^2 + qxy + rx(x^2 + y^2)$
	$\eta = A'x + B'y + C' + pxy + qy^2 + ry(x^2 + y^2)$
10...	$\xi = Ax + By + C + px^2 + qxy + ry^2$
	$\eta = Ay - Bx + D + p'x^2 + q'xy + r'y^2$
13...	$\xi = Ax + By + C + px^2 + qxy + ry^2 + sx(x^2 + y^2)$
	$\eta = A'x + B'y + C' + p'x^2 + q'xy + r'y^2 + sy(x^2 + y^2)$
14...	$\xi = Ax + By + C + px^2 + qxy + ry^2 + sx(x^2 + y^2)$
	$\eta = A'x + B'y + C' + p'x^2 + q'xy + r'y^2 + s'y(x^2 + y^2)$
20...	$\xi = Ax + By + C + px^2 + qxy + ry^2 + sx^3 + tx^2y + uxy^2 + vy^3$
	$\eta = A'x + B'y + C' + p'x^2 + q'xy + r'y^2 + s'x^3 + t'x^2y + u'xy^2 + v'y^3$

observations, and therefore they can be eliminated by the following steps. First, observe the astrometric standard regions with a large number of stars with precise positions. Second, perform a SPA with various models as, e.g., listed in Table 3 in which $N_{\rm param}$ means the number of parameters. Third, judge which model is the best according to the rms of the postfit residuals and the number of model parameters used. Usually it is the model with the least number of parameters among those of the smallest rms within the range of error. Finally, correct the (x, y) data of all CCD observations based on the results obtained from the steps above (Zacharias & Zacharias 1999).

Some terms that are not stable (i.e., that depend on the stability of the instruments system, e.g., telescope, filter and detector) should be handled for individual CCD images. Take the tangential point, for example, which is usually got from the orientation of the telescope; the biases in the coordinates (α_0, δ_0) of the tangential point introduce nonlinear terms into the relationship between the standard and measured coordinate systems as follows (Eichhorn 1974):

$$\left\{ \begin{array}{l} \xi = ax + by + c + x^2 \cos\delta_0 d\alpha_0 + xy\, d\delta_0 \\ \eta = dx + ey + f + xy \cos\delta_0 d\alpha_0 + y^2\, d\delta_0 \end{array} \right\}, \qquad (19)$$

where $d\alpha_0$ and $d\delta_0$ represent the biases in the initial coordinates (orientation of the telescope) of the tangential point from the true values. With the help of one secondary reference catalog, $d\alpha_0$ and $d\delta_0$ can be solved from equation (19) iteratively with adequate precision.

From the analysis above, a suitable secondary reference catalog with reasonable precision and densification is indispensable for calculation of $\Delta\boldsymbol{\rho}_1$ and $\Delta\boldsymbol{\rho}_2$, correction of proper motion of common stars in equation (12), and modification of the initial tangential points. Among the available, or to be available, astrometric catalogs, either TYCHO-2 (Høg et al. 2000) with $\sigma_{\alpha,\delta} = 10$–$100$ mas, $\sigma_{\mu\alpha,\mu\delta} \approx 2.5$ mas yr^{-1} and a mean density of 60 per square degree or USNO CCD Astrograph Catalog (UCAC; Zacharias et al. 2000, 2004) with $\sigma_{\alpha,\delta} = 15$–$70$ mas, $\sigma_{\mu\alpha,\mu\delta} = 1$–$7$ mas yr^{-1}, and a mean density greater than 1000 deg^{-2} could meet BA requirement for CCDs with different FOVs.

4. RESULTS OF BA SIMULATIONS

Twenty-five frames of CCD observations with the linear model are simulated as in § 3.1. The CCD frames are within a square and overlapped in the manner of corner to center as shown by Figure 2. Among the 25 CCD frames, 16 are on the top (*solid line*) and the rest nine are on the bottom (*dotted line*). Both SPA and BA are performed, and the results are compared for four different cases. Similarly,

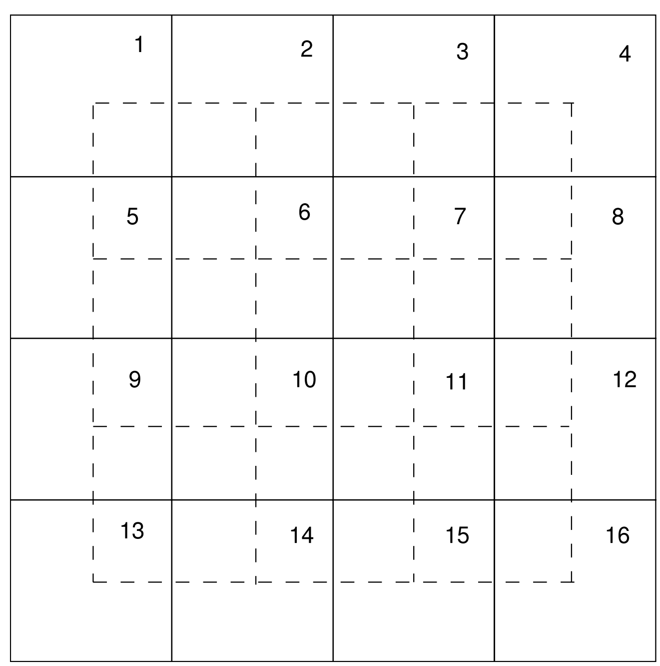

FIG. 2.—The 25 simulated CCD frames overlapped in the pattern of corner-to-center, with 16 CCD frames on the top sheet (*solid line*) and 9 on the bottom sheet (*dotted line*).

TABLE 4
Results of BA of Simulated Observations for Four Different Cases

		BA (mas)				SPA (mas)			
Case[a]	$N_{\rm ref}$[b]	$\overline{\lvert\Delta\alpha\rvert}$	$\sqrt{\sigma^2_{\Delta\alpha}}$	$\overline{\lvert\Delta\delta\rvert}$	$\sqrt{\sigma^2_{\Delta\delta}}$	$\overline{\lvert\Delta\alpha\rvert}$	$\sqrt{\sigma^2_{\Delta\alpha}}$	$\overline{\lvert\Delta\delta\rvert}$	$\sqrt{\sigma^2_{\Delta\delta}}$
1......................	16	8.1	45.2	11.7	46.6				
		8.1	34.0	11.8	36.0				
	8	13.8	50.5	14.1	50.0				
		13.9	40.3	14.3	39.8				
	4	19.3	52.6	14.3	51.9				
		19.5	42.8	14.4	42.3				
2......................	15 + 4	15.3	40.7	9.4	39.8	39.1	74.7	40.9	58.5
		14.8	28.0	10.4	26.3	39.9	68.6	40.8	51.4
3......................	15 + 4	20.5	42.5	15.9	40.4	43.7	65.2	14.5	44.3
		20.3	29.4	14.9	26.5	43.4	56.2	13.9	32.0
4......................	15 + 4	97.1	40.5	15.9	40.4	191.4	46.4	14.5	44.3
		96.7	27.1	14.9	26.5	191.0	34.7	13.9	32.0

[a] (1): Case of few reference stars. (2): Case of nonuniformally distributed reference stars. (3): Case of single reference star with significant position bias. (4): Case of regional reference stars with systematic position biases.
[b] The expression $m + n$ means there are n reference stars on the sixth CCD frame and m reference stars on the other CCD frames of the top sheet, as shown in Fig. 2.

20 simulations with different random numbers are carried out for each case.

4.1. *Case of Few Reference Stars*

BA is performed with a different number of reference stars, which are distributed evenly over the whole region of the observations. The corresponding statistical quantities of 20 simulations are listed in Table 4 (case 1) for the number of reference stars as 16, 8, and 4. It is shown that even in the extreme situation with only four reference stars on the 25 CCD frames, results with acceptable precision can be obtained by BA, while SPA of each CCD frame is unavailable because of the lack of reference stars.

4.2. *Case of Nonuniformly Distributed Reference Stars*

Suppose there are four reference stars distributed nonuniformly on the sixth CCD frame and there is one reference star on each of the other CCD frames on the top sheet. So for the sixth CCD, SPA could be performed, but the resulting LRF contains obvious errors listed in Table 4 (case 2), whereas the results of BA are better by a factor of more than 5. It is clear that, because of the enlargement in the sky coverage of reference stars, BA could improve LRF remarkably.

4.3. *Case of Single Reference Star with Significant Position Bias*

Suppose that four reference stars are evenly distributed on the sixth CCD but one of them is biased by 0″.2 in right ascension. SPA of the sixth CCD and BA of all the CCD frames are performed, and the results are listed in Table 4 (case 3). It can be seen that the position bias of an individual reference star leads to a distortion of the LRF, but it is obviously cured by BA and the results are better by a factor of 4 with respect to the SPA case.

4.4. *Case of Regional Reference Stars with Systematic Position Biases*

Let the four reference stars on the sixth CCD be biased in right ascension by an increase of 0″.2. Results from SPA of the sixth CCD and BA of all the CCD frames are listed in Table 4 (case 4). A conclusion similar to the last case can be deduced: that BA can cure the distortion of LRF, and that the results are better by a factor of 2 than the SPA case.

5. BA OF CCD OBSERVATIONS

In this section BA is further examined using CCD observations. There are two sets of CCD observations from the Xinglong station of the National Astronomical Observatories of China (NAOC). One was taken by the 60/90 cm Schmidt telescope on 2002 May 31; the other was taken by the 2.16 m telescope on 2003 March 24. The CCD FOV of the Schmidt telescope is $1^\circ \times 1^\circ$ with 2048×2048 pixels, the scale is 0.024 mm pixel^{-1}, and there are about 300 field stars. For the CCD of the 2.16 m telescope, the corresponding parameters are $10' \times 10'$, 2048×2048 pixels, 0.015 mm pixel^{-1}, and 30 field stars. It is worth noting that the CCD camera of the 2.16 m telescope is a Beijing Faint Object Spectrograph and Camera system with a focus reducer. The objects of both sets of observations are the astrometric calibration region (ACR) (Stone et al. 1999). Because the precision of coordinates and proper motions of ACR stars are quite good, 26 mas and 6 mas yr^{-1}, respectively, and the star density is as high as 1000 deg^{-2}, the observations are ideal for examining of BA.

5.1. *Preparation of the Data for BA*

The preparation of the data of the two CCD observations are handled as in § 3.3. The rms of the differences between the measured and catalog coordinates of objects versus the number of model parameters are shown Figure 3 in which the asterisks and the circles represents the Schmidt telescope and the 2.16 m telescope, respectively. For the CCD of the Schmidt telescope, the rms of the six-parameter model is about 50 mas, which is on the same level as that of the nonlinear models and indicates that the CCD model of the Schmidt telescope is linear. While for the CCD of the 2.16 m telescope, the rms (about 180 mas) of linear models is 3 times of that of nonlinear models (about 65 mas), which clearly indicates the CCD model of the 2.16 m telescope is

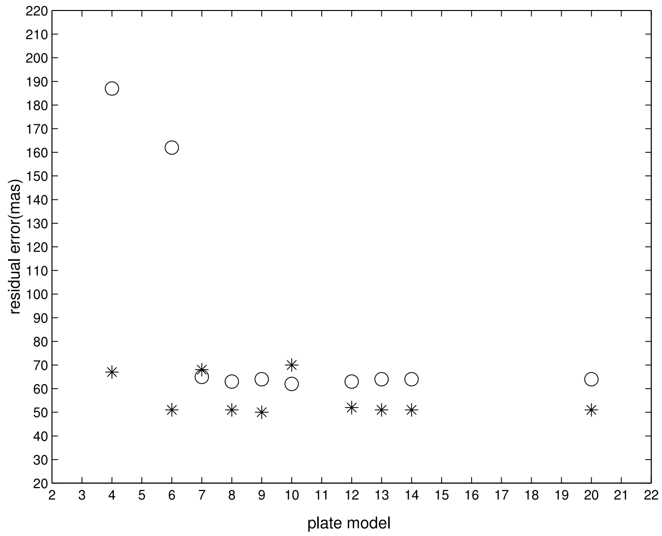

FIG. 3.—The rms of the differences between the measured and catalog coordinates of reference stars vs. the number of model parameters. The asterisks represent results of the Schmidt telescope; the circles represent those of the 2.16 m telescope.

nonlinear. It is easy to see that the best model for the CCD of the 2.16 m telescope is the eight-parameter model referred to in Table 3. According to the rule presented in § 3, all CCD data of the 2.16 m telescope are corrected before performing BA.

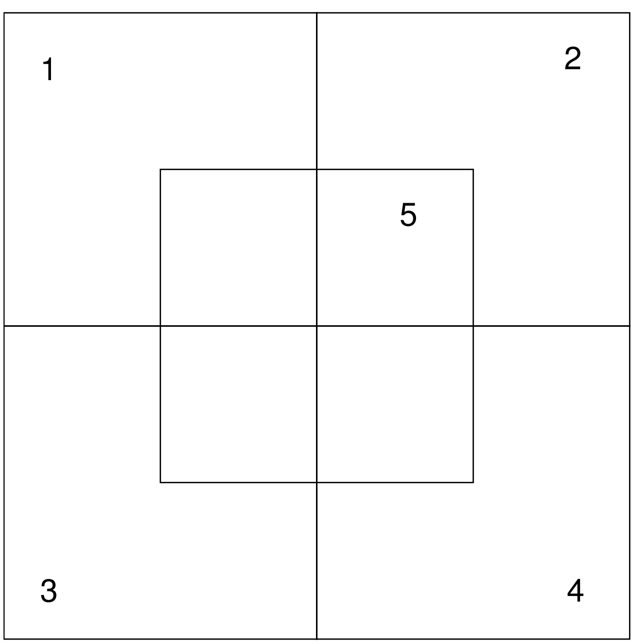

FIG. 4.—Overlapping pattern of the CCD observations.

TABLE 5
RESULTS OF BA AND SPA FOR CCD OBSERVATIONS OF SCHMIDT AND 2.16 m TELESCOPES

Telescope	N_{ref}[a]	$\overline{\Delta\alpha \cos\delta}$ (mas)	$\sigma_{\Delta\alpha}$ (mas)	$\overline{\Delta\delta}$ (mas)	$\sigma_{\Delta\delta}$ (mas)
Schmidt	0 + 4	−23	84	19	53
	4 + 4	11	50	2	64
	8 + 4	−10	50	−17	51
	12 + 4	−6	50	−19	51
2.16 m	0 + 4	−148	167	−79	109
	4 + 4	−49	81	65	73
	8 + 4	−40	78	16	65
	12 + 4	−18	67	−28	63

[a] The expression $m + n$ means there are n reference stars on the fifth CCD frame and m reference stars on the other CCD frames, as shown in Fig. 4.

5.2. *Results of BA of the CCD Observations*

The overlapping pattern of the observations is shown by Figure 4. BA and SPA are performed to both sets of observations, and some statistics of the objects on the fifth CCD are listed in Table 5. In Table 5, the two lines with "0+4" reference stars are the results of SPA for the two telescopes, respectively. It is shown that for the CCD observations of the two telescopes the results of BA of all CCD frames are better by a factor of 3 to 6 than those of SPA.

6. CONCLUDING REMARKS

Our investigation of simulated data and CCD observations show that applying BA procedures improves the LRF effectively in the case of sparse reference stars, nonuniformly distributed reference stars, a reference star with significant position bias, and regional reference stars with systematic position biases.

Overlapping observations of CCD with small FOVs can cover large areas of the sky up to full sky coverage, which in principle can be reduced with BA methods. Applications of BA are the radio-optical reference frame link, densifications of star catalogs, proper motion investigations, and kinematic studies of open clusters and solar system objects.

The observations used in this paper were made by the 2.16 m telescope operated by the Joint Laboratory for Optical Astronomy of Chinese Academy of Sciences and the 60/90 cm Schmidt telescope at the Xinglong station of the NAOC. We are thankful to Xu Zhou and Zhenyu Wu and the staff of Xinglong Station for their help when the observations were taken. We wish to thank Ronald Stone and his colleagues for providing the ACRs. We are grateful to Norbert Zacharias for his valuable comments and suggestions, which have helped to improve the paper. This work is supported by the Chinese Academy of Sciences (KJCX2-SW-T1) and the Chinese National Natural Science Foundation (10333050, 10373021).

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