Orbital Phase Dependence of Globular Cluster's Tidal Radii *

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The orbits for a sample of 45 galactic globular clusters are calculated using positions and spatial velocities based on recent compilations as well as new measurements of their absolute proper motions. The perigalactic positions of each cluster are used to determine their theoretical tidal radii in the given galactic model. The orbital phase dependence between the theoretical and observed tidal radii is evidenced.

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As the very old but not the oldest objects in the galaxy, globular clusters are the cornerstone for our understanding of the formation, structure, and dynamics of our galaxy.[1] It has been recognized for a long time that globular clusters must have a finite edge imposed by the Galactic tidal field. Most observed surface density profiles of globular clusters can be predicted by the King models;[2] the edge of a globular cluster has traditionally been estimated using the model-predicted value of King's tidal radius r_t. The classical theory has pointed out that present observed tidal radii of globular clusters are obtained when the galactic tidal force on clusters is the strongest at the perigalactic position.[3,4] The tidal radius of a cluster is taken as the corresponding Lagrangian point where the galactic force equalizes exactly the gravitational force of the cluster. Both theory [5] and numerical model computation [6,7] suggested that the tidal radius formula of von Hoerner and King [3,4] may be too large for a given perigalactic distance. Keenan's [6,7] simulations of clusters in orbit around a point-mass model galaxy indicated that King's formula for the theoretical tidal radius should be reduced by a factor 2/3. The difference arises from the elongation of the limiting tidal surface along the line between the cluster centre and the galactic centre. King's tidal radius actually refers to the distance along this axis from the cluster centre to the analogue of the inner Lagrangian point in the elliptic restricted three-body problem for a point-mass galaxy. In term of the tidally distorted equipotential surfaces in the restricted three-body problem, the identified tidal radius corresponds to the semiminor axis of the tidal surface. The semiminor axis in turn is just 2/3 of the distance from the cluster centre to the inner Lagrangian point, as noted above.[5]

Based on the classical King tidal radii theory, Peterson attempted to put constraints on the shapes of globular cluster orbits using the published values of r_t.[8] The results of this work are not entirely credible since the inferred perigalactic distances for several clusters are found to be greater than their present Galactocentric distances. Innanen *et al.*[5] carried out a similar study, but their attempts to determine individual cluster perigalactic distance were defeated.[5]

The classical tidal radius theory does not include the internal dynamical processes caused by encounters between cluster stars, the two-body relaxation.[9] Recent N-body simulations have evidently illustrated that the two-body relaxation can drive the expansion of cluster liming radius.[10] Using a hybrid numerical method, Oh and Lin found that, for short relaxation time, due to the two-body relaxation process effect, the theoretical tidal radius corresponds to the apogalactic position.[11] Moreover, the classical theory dose not take into account the energy input caused by shocks when clusters cross through the disk or the central bulge.[9] The results of Fokker–Planck models of the evolution of the globular clusters proved that the tidal shocks of bulge and disc are important for the evolution of clusters.[12] The tidal shocks affect the structure evolution of globular cluster via the following processes. First, the cluster stars gain energy, after a short period of contraction immediately following the shock, the cluster as a whole expands. Second, tidal shocks induce a dispersion of stellar energies, named the "tidal shock relaxation" which is very similar to the diffusion in energy space due to two-body relaxation.[13] Recent observation of the tidal tails around the globular cluster Palmar 5 also indicated the effect of strong shocks on the cluster structure.[14]

According to the above discussions, if the tidal radius of a globular cluster is imposed at perigalactic position just as classical tidal radius theory has pointed out, due to the internal and external dynamical effect, this tidal radius will expand along the orbit of cluster, i.e. there will be a orbital phase dependence between theoretical tidal radii at perigalactic position and present observed tidal radii. In order to calculate the theoretical tidal radii of globular clusters, the perigalactic distances must be known. Thus the orbits of clusters in a galactic model are needed. Meziane and Colin selected 21 globular clusters with known space velocities and positions, and then calculated their orbits as well as the corresponding theoretical tidal radii,[15] and found that there was a discrepancy between theoretical and observed tidal radii.

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They showed that this difference depends on the relative orbital position of the cluster, leading to an orbital phase dependence. Here our main motivation is to check the orbital phase dependence between theoretical and observed tidal radii of globular clusters using a enlarged sample of 45 clusters.

The main limit to calculate the orbits of clusters is the missing information for the absolute proper motions. Recent progress in the field of absolute proper motions of globular clusters has significantly enlarged the sample of well-measured clusters to 45, compared with the 21 clusters analysed by Meziane and Colin. The uncertainties of orbits mostly also come from the errors in absolute proper motions of clusters.[16] In order to calculate the orbits, the parameters of clusters are adopted as follows: 45 absolute proper motions of clusters come from the compilation of Wu *et al*;[17] coordinates, distances from sun, absolute V magnitudes and radial velocities are taken from the compilation of Harris.[18] Local standard of rest (LSR) velocities are determined adopting the solar motion ($10.0\,\mathrm{km\,s^{-1}}$, $5.2\,\mathrm{km\,s^{-1}}$, $7.2\,\mathrm{km\,s^{-1}}$) from Binney and Merrifield.[19] Velocities with its origin at the Galactic centre are also determined by adopting a solar Galactocentric distance 8.0 kpc and a rotation velocity of the LSR $220.0\,\mathrm{km\ s^{-1}}$. The mass-to-light ratios of clusters M/L_v, which are used to calculate the masses of clusters, come from the compilation of Pryor *et al.*.[20] Present observed tidal radii of clusters mostly are taken from the compilation of Trager *et al.*,[21] but some data are replaced with those obtained by detailed studies with more modern methods.[22–25]

Due to the fact that the particular choice of Galactic model has only negligible incidence on the results,[15] we take a simple galactic mass model to integrate the orbits of clusters:

$$M_g(R_g) = V_c^2 R_g/G,$$

where G is the constant of gravitation, $V_c = 220.0\,\mathrm{km\ s^{-1}}$ the circle velocity, R_g the Galactocentric distance. The potential is

$$\phi(R_g) = V_c^2 \ln R_g.$$

The integration routine is a Bulirsch–Stoer method with adaptive step size,[26] and the integration time is 13 Gyr. The relative change in the total orbital energy over the integration time is of the order of 10^{-11} for our adopted Galactic model.

The theoretical tidal radius of a globular cluster r_{te} is determined using the formula of Innanen:[5]

$$r_{te} = \frac{2}{3}[1 - \ln(R_p/A)]^{-1/3}\left(\frac{m}{2M_p}\right)^{1/3} R_p,$$

where R_p is the perigalactic distance of cluster, m is the mass of cluster, $M_p = M_g(R_p)$ is the mass of the galaxy within R_p, and A is a distance as the radius of a circular orbit which has the same total orbital energy for any given cluster orbit. We use the last perigalactic crossing to compute the theoretical tidal radius r_{te}.

Our simple galactic model reveals that the tidal force of the Galaxy increases with decreasing distance from the galactic centre. In the meantime, one of the important assumptions of theoretical tidal radius is that the tidal radius is imposed by the maximum of galactic tidal field along its orbit, then the cluster tidal radius should be a function of the perigalactic distance. Figure 1 gives a comparison of present observed tidal radii of clusters r_t and their perigalactic distances R_p.

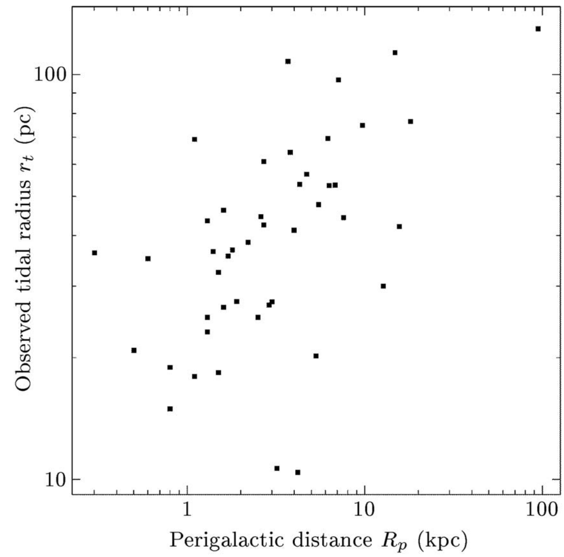

Fig. 1. Observed tidal radii r_t versus perigalactic distances R_p.

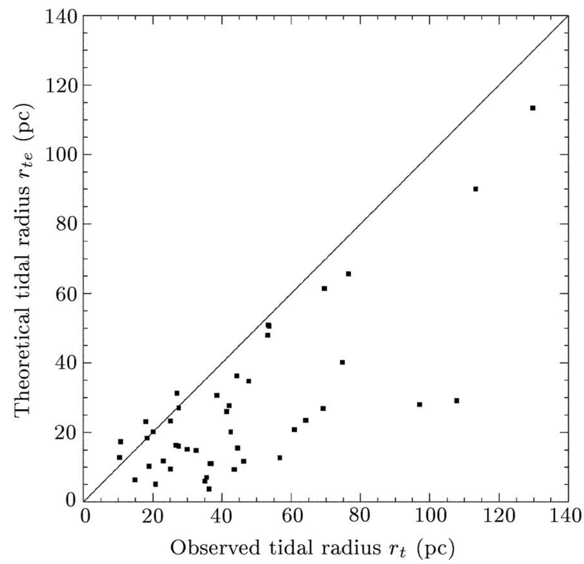

Fig. 2. Theoretical tidal radii r_{te} versus observed tidal radii r_t.

Besides a large spread in the data a correlation is clearly visible in the logarithmic figure. The linear Pearson correlation coefficient of the logarithms between r_t and R_p is 0.54, corresponding to a probability of correlation of 99.986%. The relation between r_t and R_p is close to $r_t \propto R_p^{0.3}$. This relation is very close to the mean trend of observed tidal radii along with present galactocentric distances R_g: $r_t \propto R_g^{1/3}$.[27]

These findings support to the assumption that the tidal radii of globular clusters are indeed induced by the tidal force of the Galaxy.

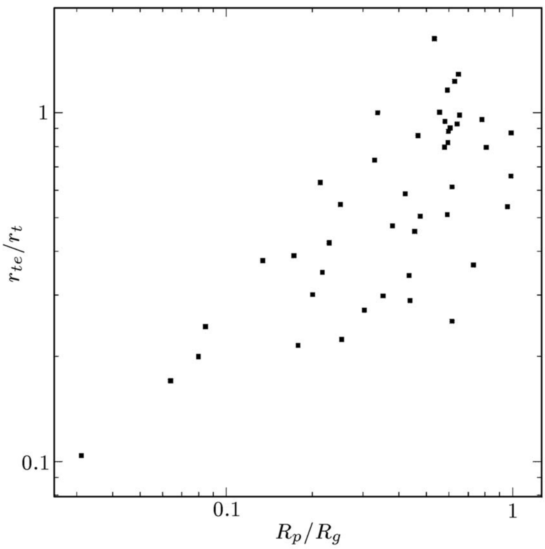

Fig. 3. Theoretical and observed tidal radii ratio r_{te}/r_t versus perigalactic and present distances ratio R_p/R_g.

We also compare the theoretical tidal radii obtained at the perigalactic distances with their observed counterparts. The results are given in Fig. 2, where the straight line represents the equality between theoretical and observed values. Figure 2 shows that, for most of the clusters, the theoretical values are small compared with the observed ones. This trend has also been noted by Meziane and Colin,[15] although they used different absolute proper motions data. The present result confirms that such a tendency is not due to the errors of proper motions and distances. Thus the theoretical tidal radius that is calculated at perigalactic distance gives an underestimated value of the observed one. We also calculate the tidal radii at the present positions R_g of the clusters, and find that the theoretical values are greater than the observed ones. At the apogalactic distances, computed values are consequently larger. These trends are consistent with what Meziane and Colin found.[15] Thus the tidal radius of a cluster is not only determined by the extent of the tidal field of galaxy when it orbits away from the perigalactic position, but is also affected by other dynamical processes.

Now, we check whether the difference between theoretical tidal radii at perigalactic distances and observed ones depends on the orbital phase which Meziane and Colin had pointed out. Following Meziane and Colin,[15] we also define the difference between theoretical and observed tidal radii by introducing r_{te}/r_t and present these ratios in terms of the radios between perigalactic and present distances R_p/R_g in Fig. 3. Unlike Figs. 4(a) and (b) of Meziane and Colin, the axis of Fig. 3 is plotted in logarithms. A general trend appears from this figure showing dependence of the tidal radius with orbital phase. When a cluster is near its perigalactic position, the theoretical tidal radius gives a good estimation of its observed value. The linear Pearson correlation coefficient of the logarithms between these two ratios is 0.74, corresponding to a probability of correlation of 99.99999%.

Table 1. Data by computing the mean of $\log(r_{te}/r_t)$ for equal intervals of $\log(R_p/R_g)$. The last column gives the number of clusters in each bin.

$\log(R_p/R_g)$	$\langle\log(r_{te}/r_t)\rangle$	n
−1.51 to −1.29	−0.98	1
−1.40 to −1.18	−0.73	2
−1.29 to −1.08	−0.52	2
−1.18 to −0.97	−0.45	5
−1.08 to −0.86	−0.36	7
−0.97 to −0.76	−0.17	15
−0.86 to −0.65	−0.14	12

In order to see the phase dependence more clearly, we smooth out the data by computing the mean of $\log(r_{te}/r_t)$ for equal intervals of $\log(R_p/R_g)$. The result is listed in Table 1 It is clear that the logarithmic ratios of theoretical and observed tidal radii depend linearly on the logarithmic ratios of perigalactic and present distances. The relation between these two ratios is close to $r_{te}/r_t \propto (R_p/R_g)^{0.6}$.

Thus our results obtained from an enlarged sample of globular cluster evidences that the ratio of theoretical and observed tidal radii depends on the relative orbital position of the cluster, i.e. an orbital phase dependence which was first found by Meziane and Colin with a smaller sample. Although the phase dependence found here is not the same as that of Meziane and Colin, $r_{te}/r_t \propto 1.1R_p/R_g$, the tidal radius phase dependence found in these two studies is very clear and probably due to internal and external dynamical processes of globular clusters.[15]

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