CHIN. PHYS. LETT. Vol. 26, No. 2 (2009) 029701

Mass of Open Cluster NGC 7789 *

WU Zhen-Yu(武振宇)[1**], DU Cui-Hua(杜翠花)[2], MA Jun(马骏)[1], ZHOU Xu(周旭)[1]

[1] *National Astronomical Observatories, Chinese Academy of Sciences, Beijing 100012*

[2] *College of Physical Sciences, Graduate University of the Chinese Academy of Sciences, Beijing 100049*

(Received 19 September 2008)

Mass of open cluster NGC 7789 in the Galaxy is determined by three methods: the photometric method based on mass-luminosity relations of stars in the cluster $M_{\rm pho} = 7712.5M_\odot$; the dynamical method based on virial theorem $M_{\rm vir} = 6996.1M_\odot$; and the tidal radius method based on the interaction between the cluster and the Galaxy $M_{\rm tid} = 5152.5M_\odot$. The mean mass of this cluster is estimated to be $M_c = 6620.4 \pm 762.5M_\odot$.

PACS: 97.10.Xq, 98.20.Di

Mass is one of the fundamental parameters of star clusters. Open clusters, only with enough initial masses, can survive various internal and external evolution processes and be observed in the solar neighbourhood at present.[1,2,3] There are three independent methods for estimating cluster masses.

The first method is the simplest and most straightforward way, which is to count cluster members and to sum up their masses. The stellar mass-luminosity relation is used in this method. The cluster mass derived by this method can be defined as "photometric mass" $M_{\rm pho}$. This method is limited by the limiting magnitude and by the limited area covered by a study. The extrapolation of the mass spectrum to an unobserved lower limit of stellar masses is frequently applied in this method.

The second method is to apply the virial theorem, which dictates that for a cluster in equilibrium, its mass can be determined by $M_c \propto \sigma^2$. Here σ is the stellar velocity dispersion in the cluster. This method is limited by the accuracies of the derived velocity dispersions. At present, most of the velocity dispersions for open clusters are determined based on radial velocities, and only few can be derived from proper motion data.[4,5] The velocity dispersions derived from radial velocities are also affected by the binary stars in the cluster, which will overestimate the cluster mass.[6] Cluster mass determined by this method can be defined as 'virial mass' $M_{\rm vir}$.

The third method uses the interpretation of the tidal interaction of a cluster with the Galaxy, and the tidal radius of a cluster is needed.[4] If the tidal radius r_t of a cluster is known, the mass of this cluster can be determined by $M_c \propto r_t^3$. This method is limited by the accuracies of the derived tidal radii r_t. Cluster mass determined by this method can be defined as 'tidal mass' $M_{\rm tid}$.

In this Letter, using the three methods mentioned above, we determine the mass of open cluster NGC 7789 in our Galaxy. The open cluster NGC 7789 has been studied with numerous photometric and spectral observations because of its very rich stellar population. Recently, using the Beijing–Arizona–Taiwan–Connecticut (BATC) Multi-Colour Survey photometric system, Wu *et al.* (W07)[7] derive the fundamental parameters for this cluster based on 13 band CCD intermediate-band photometry with a field of view $58' \times 58'$. By comparing observed spectral energy distributions (SED) of member stars with theoretical ones, W07 obtained the fundamental parameters for NGC 7789: an age of 1.4 ± 0.1 Gyr, a distance modulus $(m - M)_0 = 11.27 \pm 0.04$, a reddening $E(B - V) = 0.28 \pm 0.02$, and a metallicity with the solar composition.

In order to derive the 'photometric mass' $M_{\rm pho}$ of NGC 7789, all of member stars of this cluster should be observed, which requires deep and large field of view CCD photometry. The photometry of W07 can cover NGC 7789 and extends to its tidal radius. More important, using the SED-fitting method,[8] W07 can determine the membership probability for each star in this cluster which can be used to reject most of field stars and is important to derive mass function (MF) of member stars in the cluster. For member stars with masses from 0.95 to $1.85M_\odot$, the MF is fitted with a power-law function $\phi(m) \propto m^\alpha$.[7] For the given mass range of member stars, the 'photometric mass' $M_{\rm pho}$ of the cluster can be derived as

$$M_{\rm pho} = \int_{m_{\rm low}}^{m_{\rm up}} m\,\phi(m) dm, \quad (1)$$

where $m_{\rm low}$ and $m_{\rm up}$ are lower and upper mass limits of member stars. Here $m_{\rm up} = 1.85M_\odot$ is the mass of the brightest main-sequence star in this cluster. Limited by the limiting magnitude and photometric incompleteness, W07 can only derived reli-

*Supported by the National Natural Science Foundation of China under Grant Nos 10473012, 10573020, 10633020, 10673012, 10603006, 10778720, 10873016 and 10803007, the National Basic Research Programme of China under Grant No 2007CB815403, and the Knowledge Innovation Program of the Chinese Academy of Sciences.

**Email: zywu@bao.ac.cn

able MF for stars with masses larger than $m_{\rm low} = 0.95M_\odot$. The MF of member stars should be extrapolated to the H-burning mass limit of $0.08M_\odot$ to estimate the total $M_{\rm pho}$. The universal initial mass function[9] is used to extrapolate the MF for stars with masses less than $0.95M_\odot$. For stars with masses in the range $0.50 \le m < 0.95M_\odot$, the power exponent $\alpha = -2.3$ is adopted. For stars with masses in the range $0.08 \le m < 0.50M_\odot$, the power exponent $\alpha = -1.3$ is adopted. The total $M_{\rm pho}$ can be estimated to be $7712.5M_\odot$. If only stars with masses larger than $0.5M_\odot$ are considered, $M_{\rm pho}$ is $5084.8M_\odot$. If only stars with masses larger than $0.95M_\odot$ are considered, $M_{\rm pho}$ is $2113.1M_\odot$.

The virial mass $M_{\rm vir}$ can be estimated using the equation derived by Spitzer:[10]

$$M_{\rm vir} = \eta \frac{r_h \sigma_c^2}{G}, \tag{2}$$

where G is the gravitational constant, η is a dimensionless constant, r_h is the half-light radius of the cluster, and σ_c is the line-of-sight velocity dispersion. W07 fitted the surface density profile of NGC 7789 with the empirical King model:[11]

$$f = k\left\{\frac{1}{[1+(r/r_c)^2]^{1/2}} - \frac{1}{[1+(r_t/r_c)^2]^{1/2}}\right\}^2, \tag{3}$$

where r_c is the core radius, r_t is the tidal radius, and k is a normalization factor. Using stars with membership probabilities greater than 1% and with limiting magnitude $B \sim 19.0$ mag, W07 derived $r_c = 7.69$ arcmin and $r_t = 29.48$ arcmin. The derived tidal radius of NGC 7789 is close to the limiting radius can be covered by the photometric data of W07 which may underestimate the derived tidal radius. To check the affect of the limited area covered by the BATC observation, we use the USNO-B catalog data to derive the structure parameters of NGC 7789 with King model.

USNO-B[12] is an all-sky catalog that presents positions, proper motions, magnitudes in various optical bands for 1042618261 objects. The data were obtained from scans of 7435 Schmidt plates taken for the various sky surveys. USNO-B is complete down to $V = 21$ mag. The Aladin[13] tool was used to extract stars in a circular area with a radius of 45′ centred on NGC 7789. To determine the radial surface density, the stars with limiting magnitudes of 20.0 in the USNO-B $B2$ band are divided into a number of concentric rings from the centre of the cluster with a step of 1.0′.

Figure 1 shows the radial surface density profile of NGC 7789 for stars with limiting magnitudes of 20.0 in the USNO-B $B2$ band. The open circles indicate the stellar densities in each concentric circle which are obtained by dividing the number of stars in each annulus by its area. The error bars represent $1\,\sigma$ Poisson errors for each bin. Figure 1 indicates that the radial surface densities of NGC 7789 decrease along the radii from the centre and become flat beyond the radius of 25′ where the field stars dominate. The field-star contamination is estimated with the average surface density for stars in the rings at $30' < r < 40'$. We fitted the empirical King model to our field-subtracted radial density profiles. The fit was performed using a nonlinear least-squares routine that used the $1\,\sigma$ Poisson errors as weights. The best-fitting King model parameters are $r_c = 8.82' \pm 0.91'$ and $r_t = 32.20' \pm 2.80'$. Our results are consistent, within the errors, with that derived by W07. The half-light radius $r_h = 8.81'$ is also derived which is close to the derived core radius for this cluster.

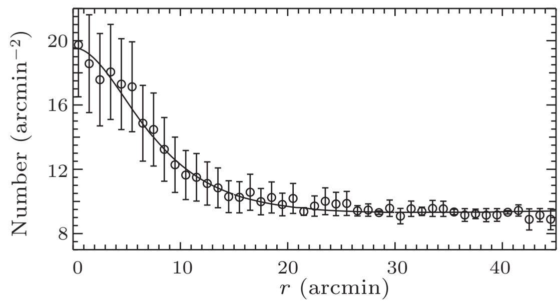

Fig. 1. Radial surface density profile of NGC 7789 for stars with limiting magnitudes of 20.0 in the USNO-B $B2$ band. The solid line shows the best-fitting King model to the radial profile, and error bars represent $1\,\sigma$ Poisson errors.

Rejecting the minimum 11.0 and maximum 11.7, we averaged the distance moduli listed in Table 1 of W07 and obtain the mean of $(m-M)_0$ to be 11.37 ± 0.1, which is used to scale the structure parameters from arcmin to parsec. Using the distance 1879.3 ± 86.6 pc derived from the above averaged distance modulus, the derived King model parameters in parsecs are $r_c = 4.8 \pm 0.5$ pc and $r_t = 17.6 \pm 1.7$ pc.

Gim *et al*(G98)[14] observed a total of 597 radial velocities for 112 stars in NGC 7789. The radial velocities of 50 stars identified by G98 as members with constant velocity indicating they are single stars are used to derive the mean velocity v_c and the velocity dispersion σ_c of the cluster. We use the methods from Pryor and Meylan[15] to determine v_c, σ_c, and their errors. First, we assume that the velocity distribution is the normal function:

$$f(v_i) = \frac{1}{[2\pi(\sigma_c^2+\sigma_i^2)]^{1/2}} \exp\left[-\frac{(v_i - v_c)^2}{2(\sigma_c^2+\sigma_i^2)}\right], \tag{4}$$

where σ_i is the error for the observed radial velocity v_i for each star. The parameters v_c and σ_c can be estimated by the maximum likelihood method which leads to the following two equations:

$$\sum_{i=1}^{N} \frac{v_i}{(\sigma_c^2+\sigma_i^2)} - v_c \sum_{i=1}^{N} \frac{1}{(\sigma_c^2+\sigma_i^2)} = 0, \tag{5}$$

$$\sum_{i=1}^{N}\frac{(v_i-v_c)^2}{(\sigma_c^2+\sigma_i^2)^2}-\sum_{i=1}^{N}\frac{1}{(\sigma_c^2+\sigma_i^2)}=0, \quad (6)$$

where N is the number of observed stars. These equations can be solved numerically. The errors σ_{v_c} and σ_{σ_c} for v_c and σ_c can be estimated as:

$$\sigma_{v_c}^2 = A_{22}/(A_{11}A_{22}-A_{12}^2), \quad (7)$$

$$\sigma_{\sigma_c}^2 = A_{11}/(A_{11}A_{22}-A_{12}^2), \quad (8)$$

where

$$A_{11}=\sum_{i=1}^{N}\frac{1}{(\sigma_c^2+\sigma_i^2)}, \quad (9)$$

$$A_{22}=\sum_{i=1}^{N}\Big[\frac{1}{(\sigma_c^2+\sigma_i^2)}-\frac{(v_i-v_c)^2+2\sigma_c^2}{(\sigma_c^2+\sigma_i^2)^2}+\frac{4\sigma_c^2(v_i-v_c)^2}{(\sigma_c^2+\sigma_i^2)^3}\Big], \quad (10)$$

$$A_{12}=\sum_{i=1}^{N}\frac{2\sigma_c(v_i-v_c)}{(\sigma_c^2+\sigma_i^2)^2}. \quad (11)$$

For NGC 7789, using the data of G98, the mean velocity and the velocity dispersion as well as their errors can be estimated as $v_c = -54.91 \pm 0.12\,\mathrm{km\,s^{-1}}$ and $\sigma_c = 0.80 \pm 0.09\,\mathrm{km\,s^{-1}}$.

Using the derived r_c and σ_c, and the Spitzer's equation with the constant $\eta = 9.75$,[6] the virial mass of NGC 7789 can be determined as $M_{\rm vir} = 6996.1 \pm 807.8 M_\odot$. The error is estimated by taking into account the errors in r_c and σ_c.

The dominant assumptions underlying the validity of Spitzer's equation is that the cluster is in virial equilibrium. The half-mass relaxation time t_{rh} is the time-scale for a cluster to reach to dynamical equilibrium and can be determined by[10]

$$t_{rh} = 0.138\frac{M_c^{1/2}\, r_h^{3/2}}{\langle m\rangle\, G^{1/2}\, \ln(0.4n)}, \quad (12)$$

where $\langle m\rangle$ is the mean mass of stars and n is the total number of stars in the cluster which can be determined from the MF: $n = \int_{m_{\rm low}}^{m_{\rm up}} \phi(m)dm$. Using the derived 'photometric mass' $M_{\rm pho} = 7712.5 M_\odot$ and $r_h \simeq r_c = 4.8\,\mathrm{pc}$, the half-mass relaxation time is estimated as $t_{rh} = 516.8\,\mathrm{Myr}$, which is less than the age of 1.4 Gyr for NGC 7789. So, NGC 7789 has reached to dynamical equilibrium and Spitzer's equation can be used to determine the mass of this cluster.

Kouwenhoven and de Grijs[6] pointed out that the presence of binaries in cluster will increase the measured velocity dispersion due to the orbital motion of binary stars, and may therefore result in a dynamical mass overestimation. The adopted radial velocities for NGC 7789 are determined from single stars based on the observations with a period of 20 years, so binary stars have been rejected.[14] The effect of binary for derived velocity dispersion in the present study can be ignored.

The classical theory has pointed out that present observed tidal radii of cluster are obtained when the galactic tidal force on clusters is the strongest at the perigalactic positions. The tidal radius of a cluster is taken as the corresponding Lagrangian point where the Galactic force equalizes exactly the gravitational force of the cluster.[11] Adopting point-mass Galactic model, King derived the relation between the tidal radius and the mass of a cluster as follows:[11]

$$r_t = R_p\left[\frac{m_c}{(3+e)M_g}\right]^{1/3}, \quad (13)$$

where R_p is the perigalactic distance of cluster from the centre of the Galaxy, e is the orbital eccentricity of cluster, and M_g is the mass of the Galaxy within R_p.

Both theory[16] and numerical computation[17] indicated that the tidal radius formula of King may be too large and should be reduced by a factor of 2/3. The difference arises from the elongation of the limiting tidal surface along the line between the cluster centre and the Galactic centre. King's tidal radius value actually refers to the distance along this axis from the cluster centre to the analogue of the inner Lagrangian point in the elliptic restricted three-body problem for a point-mass Galaxy.

In fact, measuring the tidal radius of a real cluster employs the projected star density with radius from the cluster centre. The observed projected density profile of a cluster is invariably fitted to a spherical model just as what we have carried out in this study, the "observed" tidal radius just specifies the spherical equipotential at which the density vanishes. In terms of the tidally distorted equipotential surfaces in the restricted three-body problem, tidal radius identified in this way corresponds to the semiminor axis of the tidal surface. The semiminor axis in turn is just 2/3 of the distance from cluster centre to the inner Lagrangian point.[16]

On the other hand, point-mass model is too simple to represent the true mass distribution of the Galaxy. Another simple but more true mass model of the Galaxy is adopted as $M_{g(R_g)} = V_0^2 R_g/G$, where R_g is the distance from the Galactic centre, V_0 is the circular rotation speed of the Galaxy, and $M_{g(R_g)}$ is the mass of the Galaxy within R_g. The tidal radius r_t and the mass of cluster M_c has the relation[16]

$$r_t = \frac{2}{3}[1-\ln(R_p/A)]^{-1/3}\left[\frac{M_c}{2M_g(R_p)}\right]^{1/3} R_p, \quad (14)$$

where A is a distance from the centre of the Galaxy as the radius of a circular orbit which has the same total orbital energy as the elliptical orbit of a cluster. A can be estimated with $e = (1-R_p/A)$. Substituting the mass model of the Galaxy into Eq. (14), we obtain

$$M_c = \frac{27\, V_0^2\, r_t^3}{4\, R_p^2\, G}[1 - \ln(R_p/A)]. \quad (15)$$

In order to calculate the tidal mass $M_{\rm tid}$ with Eq. (15), the perigalactic distance R_p and orbital eccentricity e of the cluster should be known. Wu *et al.* (W08)[18] calculate the orbits for a sample of 347 open clusters in the Galaxy. The derived orbital eccentricities for most of clusters in their sample are less than 0.1. However, for NGC 7789, W08 derived a orbital eccentricities $e = 0.37$ and a perigalactic distance $R_p = 4.4$ kpc. Adopting a rotation velocity of $V_0 = 220.0\,{\rm km\,s^{-1}}$ and using Eq. (15), the tidal mass of NGC 7789 can be estimated to be $M_{\rm tid} = 31294.2 M_\odot$. Compared with the derived photometric mass $M_{\rm pho}$ and virial mass $M_{\rm vir}$, the derived tidal mass $M_{\rm tid}$ is too large.

The classical theory[11,16,17] considers that the tidal radius of a cluster is imposed at the perigalactic position and does not include the internal and external dynamical processes when the cluster moves along its orbit in the Galaxy. The internal two-body relaxation and the external tidal shock relaxation will expand the cluster and there will be a orbital phase dependence between the classical tidal radius imposed at the perigalactic position and the present observed tidal radius.[19]

W08 derived a orbital period of $\sim$ 200 Myr for NGC 7789 which is about half the half-mass relaxation time $t_{rh} \sim 500$ Myr. This cluster has time to expand to its present observed tidal radius in one orbital period, and in Eq. (15), R_p should be replaced by present observed position of cluster R_g. For the present position R_g, $R_g/A \sim 1$ and Eq. (15) can be rewritten as

$$M_c = \frac{27\, V_0^2\, r_t^3}{4\, R_g^2\, G}. \quad (16)$$

Adopting the solar Galactic distance $R_\odot = 8.0 \pm 0.5$ kpc and the distance of cluster from the Sun $d = 1879.3 \pm 86.6$ pc, the Galactic distance of cluster can be calculated to be $R_g = 8.97 \pm 0.49$ kpc, where the error mostly come from the estimated error of 0.5 kpc for the Solar Galactic distance. Using Eq. (16), the tidal mass of NGC 7789 can be estimated to be $M_{\rm tid} = 5152.5 \pm 1521.4 M_\odot$. Only the error 1.7 pc in r_t is considered in the estimated error $1521.4 M_\odot$ for $M_{\rm tid}$.

If we use the same point-mass model as the King model and add the correction factor 2/3 to King's formula, the tidal mass of NGC 7789 can be estimated at the present observed position as $M_{\rm tid} = 5235.9 M_\odot$, which is very close to our derived values based on different mass models of the Galaxy.

Using King's formula, with tidal radius $r_t = 19.7$ pc and the distance of cluster from the Sun $d = 2337$ pc, Piskunov *et al.* (P07)[4] estimated the tidal mass $M_{\rm tid} = 2055.89 M_\odot$ for NGC 7789. P07 did not add the 2/3 correction factor to King's formula and the derived mass of cluster was obviously underestimated. On the other hand, using the distance $d = 2337$ pc adopted by P07, the tidal radius of P07 can be determined as $r_t = 28.97$ arcmin, which is consistent with that derived by us with the USNO-B data. The adopted distance d of P07 is too large than our adopted value. The known maximum distance modulus derived for NGC 7789 is $(m - M)_0 = 11.70$ [7] corresponding to a distance $d = 2187.8$ pc, which is also less than the value adopted by P07. Using the same King's formula and tidal radius as P07 but adopting the distance $d = 1879.3$ pc, we can estimate the tidal mass for NGC 7789 to be $M_{\rm tid} = 2523.8 M_\odot$, which is about 1/5 larger than the derived value by P07. Thus the systemic uncertainties in the adopted distance of the cluster obviously affect the derived tidal mass $M_{\rm tid}$.

Using the derived photometric mass $M_{\rm pho} = 7712.5 M_\odot$, virial mass $M_{\rm vir} = 6996.1 M_\odot$, and tidal mass $M_{\rm tid} = 5152.5 M_\odot$, the mean mass of NGC 7789 can be estimate to be $M_c = 6620.4 \pm 762.5 M_\odot$.

The derived $M_{\rm pho} = 7712.5 M_\odot$ is the upper limit of the photometric mass for NGC 7789. The photometric mass is affected by the adopted lower limit of mass for stars in the cluster. The virial mass and tidal mass are all affected by the adopted distance of the cluster. However, the effect of errors in distance for the virial mass is smaller than that for the tidal mass. The relation between the tidal radius and the mass of cluster is very uncertain due to different theories.

References

[1] Wu Z Y Shu C G and Chen W P 2003 *Chin. Phys. Lett.* **20** 1648
[2] Gieles M Athanassoula E and Portegies Zwart S F 2007 *Mon. Not. R. Astron. Soc.* **376** 809
[3] Lamers H J G L M and Gieles M 2006 *Astron. Astrophys.* **455** L17
[4] Piskunov A E et al 2007 *Astron. Astrophys.* **468** 151
[5] de Grijs R et al 2008 *Mon. Not. R. Astron. Soc.* Accepted
[6] Kouwenhoven M B N and de Grijs R 2008 *Astron. Astrophys.* **480** 103
[7] Wu Z Y et al 2007 *Astrophys. J.* **133** 2061
[8] Wu Z Y et al 2006 *Publ. Astron. Soc. Pacific.* **118** 1104
[9] Kroupa P 2001 *Mon. Not. R. Astron. Soc.* **322** 231
[10] Spitzer L 1987 *Dynamical Evolution of Globular Clusters* (Princeton, NJ: Princeton University Press)
[11] King I 1962 *Astrophys. J.* **67** 471
[12] Monet D G et al 2003 *Astrophys. J.* **125** 984
[13] Bonnarel F et al 2000 *Astron. Astrophys.* **143** 33
[14] Gim M et al 1998 *Publ. Astron. Soc. Pacific.* **110** 1172
[15] Pryor C and Meylan G 1993 *ASP Conf. Ser.* **50** 357
[16] Innanen K A, Harris W E and Webbink R F 1983 *Astrophys. J.* **88** 338
[17] Keenan D W 1981 *Astron. Astrophys.* **95** 340
[18] Wu Z Y et al 2008 *Mon. Not. R. Astron. Soc.* (submitted)
[19] Wu Z Y Zhou X and Ma J 2004 *Chin. Phys. Lett.* **21** 418