Detecting Clusters of Galaxies in SDSS: I. Photometric Redshifts of Galaxies

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ABSTRACT

With the great volume of data of Sloan Digital Sky Survey (SDSS) being released, large area and complete galaxy cluster survey becomes available. As the first step of our cluster study, we present a study of photometric redshifts (PRs) of galaxies using five-band photometry obtained by SDSS. The comprehensive application of several empirical methods, such as template fitting, color-magnitude-redshift relation and quadratic correction, improves PR estimation significantly. In a test of about 460,000 galaxies with known spectroscopic redshifts (SRs), our method achieves a better result, namely that the PR uncertainty can be reduced to \( \sigma_{PR} \sim 0.022 \). The application to SDSS photometric data suggests that with the available Sloan spectroscopic sample this method is practical and effective to a limiting magnitude of \( r' \sim 21 \).

Subject headings: galaxies: clusters — galaxies: distances and redshifts — methods: data analysis

1. INTRODUCTION

The importance of galaxy clusters has been stressed by many astronomers (e.g., Dressler 1984; Bahcall 1988). A better understanding of galaxy evolution and cosmology can be directly stimulated by the detailed studies of clusters of galaxies (Bahcall & Soneira 1983; Dressler 1984; Dressler & Gunn 1992; Carlberg et al. 1996; Fan et al. 1997; Henry 2000; Blake & Bridle 2005). However, in past decades, such studies are focused mainly on nearby \((z \leq 0.1)\) clusters of galaxies listed in Abell (1958) and Abell et al. (1989). Till recent years,
people have devoted much effort to finding high redshift clusters out to $z \sim 1.2$ or higher and studying them (Postman et al. 1996; Zaritsky et al. 1997; Olsen et al. 1999; Blanton et al. 2003; Mullis et al. 2005), but the survey areas are limited within an order of $\sim 10$ square degree. Therefore, it is important to obtain a complete and deep catalog of clusters covering a large area. The SDSS (York et al. 2000) photometric data in $u', g', r', i', z'$ has been released recently, allowing us to construct a complete cluster catalog out to $z \sim 0.5$ or deeper (Bahcall et al. 2003). Several methods have been successfully applied to detect galaxy clusters in the SDSS (Kim et al. 2002; Bahcall et al. 2003; Goto et al. 2002; Miller et al. 2005).

We develop a new algorithm which will be presented in the second paper of this series (Yang et al. 2006) to detect clusters of galaxies in SDSS. Redshifts of galaxies are required in our new algorithm. SDSS has obtained the spectra of galaxies brighter than $r'_{\text{Petro}} \sim 17.77$, and down to $r'_{\text{Petro}} \sim 19.5$ for luminous red galaxies (Strauss et al. 2002; Eisenstein et al. 2001). However, the SDSS spectroscopic survey is too shallow to construct a deep and complete cluster catalog. Thus, as a part of our series works, we first study the photometric redshifts of the SDSS galaxies.

With the increasing precision, PR technique has become the most promising and essential tool for estimating redshifts of galaxies in recent studies of galaxies and cosmology (Budavári et al. 2003; Botzler et al. 2004; Babbedge et al. 2005). The first idea of estimating redshifts of galaxies from photometry dates back to Baum (1962). Since 1980’s, many investigators have studied the PR technique (Puschell et al. 1982; Spillar 1985; Loh & Spillar 1986). A review of early PR studies can be found in Koo (1999), and the perspectives and applications of PR technique are reviewed by Szalay (1999). Basically, there are two ways to estimate the PRs of galaxies, one is based on the template fitting with either theoretical or empirical spectral templates (e.g., Bolzonella et al. 2000), and the other is to establish the relationship between redshift and multi-band photometry, for instance, the investigation of quadratic fit of PR by Connolly et al. (1995), the empirical color-redshift relation for the galaxies in Hubble Deep Field (Wang et al. 1998), Bayesian PR estimation (Benítez 2000), and PR technique based on Neural Networks (Firth et al. 2003). Actually, there are several papers discussing PR technique based on Sloan data (Rojas et al. 2004; Weinstein et al. 2004; Csabai et al. 2003; Vanzella et al. 2004). The most recent work by Padmanabhan et al. (2005) gives a good performance in PR estimate for luminous red galaxies by utilizing the theoretical Simple Stellar Population (SSP) models in Bruzual & Charlot (2003). They quote an error of PR to be $\sim 0.035$ at $z < 0.55$. Other good results of PR estimation are achieved by artificial neural network (ANN) by Vanzella et al. (2004) and Collister & Lahav (2004), reaching a PR precision of $\sigma \sim 0.023$. However, ANN depends on the completeness of the training sample so strongly that it is hard to extend the PR estimation to the deep
photometric sample, e.g., to $r' \sim 21$, based on present relatively shallow spectral survey.

This paper is structured as follows. We first describe the data used for studying PR technique in Sect. 2. We attempt, in Sect. 3, to develop an empirical method based on several techniques, such as template fitting, color-redshift relation, and quadratic fitting based on photometric properties. The final result is presented in Sect. 3.3. Our method is discussed and evaluated by an application to the deep photometric data in Sect. 4. In Sect. 5 we summarize our study.

2. DATA

We find 459,584 galaxies from SDSS Data Release Four (DR4, Adelman-McCarthy et al. 2006) spectroscopic catalog, and download their five-band photometric data, redshifts, together with the Galactic extinction in each band. The Sloan spectroscopic survey of galaxies is mainly focused to $z < 0.5$ because of the limiting magnitude. In order to investigate PR estimation at higher redshift, we find additional 1,586 redshifts of galaxies with Sloan imaging observation at $z > 0.5$ from the NASA/IPAC extragalactic database (NED). Totally, we have 461,170 galaxies (hereafter sample $S$) with known SRs. The Sloan spectroscopic survey is complete to $r' \sim 18$, and for fainter galaxies the sample is dominated by red galaxies. Therefore, the sample is biased for $r' > 18$, if we use it as PR calibrating data or template. For testing our method, a catalog of 1,347,009 galaxies (hereafter sample $P$) brighter than $r' = 23$ is downloaded from the DR4, containing five-band photometry and their Galactic extinction, within a sky region defined by $162^\circ < \text{R.A.} < 202^\circ$ and $-1^\circ 25 < \text{Decl.} < 1^\circ 25$. For all of the above galaxies, the Galactic extinction is corrected before the following studies.

3. PR ESTIMATION

3.1. Color-Magnitude-Redshift Relation

Redshifts of galaxies are linked not only to their colors and spectral types but also to apparent magnitudes (Koo & Kron 1992; Koo 1999). Some works (Csabai et al. 2003; Hsieh et al. 2005) suggest that PR can be improved if we focus on a sample of galaxies with similar photometric properties, e.g., in the color-color or color-magnitude plane. Furthermore the relationship between photometric properties and redshift is intrinsically non-linear, and can not be described by simple functions. This point can be understood from Figure 1 if we plot $g'-r'-i'$ diagrams for 62,512 galaxies with $r' < 19$ in sample $S$. The redshifts of galaxies in the figure are indicated by increasing shades of gray. Although redshift appears to have
a general tendency over the color-color diagrams (CCDs), there are still some fluctuating features making the relation intricate. Especially at $z \sim 0.4$ (at about $g'-r' \sim 1.7$ and $r'-i' \sim 0.7$ in Figure 1d), the relation between color and redshift has a sharp turn, nearly perpendicular to the relation at $z < 0.4$. Further, we may notice that a given color is possibly corresponding to several obviously different redshifts which, however, can be distinguished by magnitude to some extent. By comparing the panels of Figure 1, we may find that redshift ranges of galaxies vary with apparent magnitude. In brief, all these complex features are the fundamentals to construct the empirical relation between redshift and photometric properties.

Since it is hard to find a function to describe the relation, our simple idea is to construct a matrix of redshift, colors and magnitudes by digitalizing the known relations. We first divide sample $S$ into sub-samples by magnitude, from subsample R1 to R7 whose magnitude range can be found in Table 1. We take a $g'-r'-i'$ diagram of R3 as an example of digitization. The diagram with range of $-0.25 < g'-r' < 2.3$ and $-0.25 < r'-i' < 1.2$ is divided into $400 \times 400$ bins. For each bin, we calculate the median redshift if the number of galaxies exceeds 25, otherwise we extend the region of bins till the count reaches 25 or the region radius reaches 5 bins, and then calculate median redshift of several bins that correspond to a larger color region. In fact this is a process of adaptive smoothing (Yang et al. 2004). The difference is that we do not smooth the count image, but smooth the redshift information carried by each color bin. With the mean color location and smoothed redshift of each bin, we achieve a color-redshift matrix that can be expressed as an image as the panel CMR$_{I}$-R3 of Figure 2. The same process is applied to other CCDs. Hence, we construct the empirical CMR matrices which are plotted as images in Figure 2. We call CMRI for matrices of $u'-g'-r'$, and CMR$_{II}$ for matrices of $g'-r'-i'$. For a given galaxy, according to its color and magnitude location, we can calculate the redshift from the matrices.

By sample $S$, we test the CMR matrices. Figure 3 shows the results from CMR$_{I}$ and CMR$_{II}$ respectively. CMR$_{I}$ leads to a better result at $z_{I} < 0.3$ while CMR$_{II}$ gives a good performance at $z_{II} > 0.2$. The complementary feature allows us to combine these two results. First we calculate redshifts $z_{II}$ according to CMR$_{II}$, then replace $z_{II}$ by $z_{I}$ from CMR$_{I}$ for those galaxies with $0.05 < z_{II} < 0.2$. Especially, for galaxies located in $z_{I} < 0.05$ and $z_{II} < 0.1$, we take $z_{I}$ as the results in order to partially deal with the underestimating case of $z_{II}$ (see the region of $z_{II} < 0.05$ and $z_{spec} > 0.1$ in the right panel of Figure 3). The combined redshift ($z_{CMR}$) is compared with spectroscopic redshift $z_{spec}$ in Figure 4 for 460,103 galaxies in sample $S$, suggesting an error of PR determination to be $\sigma_{CMR} = 0.0226^{1}$. It should be

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1Throughout the paper, we use the ROSTAT software (Beers et al. 1990) to calculate the biweight scale which represents a robust estimate of the dispersion ($\sigma$) for a given distribution of physical parameters,
noted that PR estimation fails if colors of galaxies are located outside the CMR matrices. The application to sample \( \mathcal{P} \) indicates that we lost about 5% estimation for limiting magnitude of \( r' = 21 \), and about 10% for \( r' = 23 \).

### 3.2. Template Fitting

Template fitting is a usual method for PR determination. It aims at fitting observed spectral energy distributions (SEDs) with the template SEDs that are calculated from synthesis models, such as GISSEL98 (Bruzual & Charlot 1993) in HYPERZ (Bolzonella et al. 2000), and SSP models (Bruzual & Charlot 2003) in Padmanabhan et al. (2005). Here, we make an attempt to use the observed SEDs with spectroscopic confirmation as template. The obvious advantages are 1) it is a real sample with accurate SED measurements for different types of galaxies at different redshifts; 2) we could avoid touching evolution and reddening effect, because these effects are imprinted in the observed SEDs. On the other hand, the disadvantages are 1) it is a challenge to extrapolate the observed template SEDs to a higher redshift; 2) a set of template SEDs is assumed to be complete in spectral type within a given redshift range, thus the result will be dependent on the template SEDs selected.

We use the following typical \( \chi^2 \)-minimization equation

\[
\chi^2 = \sum_{i=1}^{n} \left[ \frac{p_i^{\text{obs}} - p_i^{\text{temp}} - c_0}{\sigma_i} \right]^2 ,
\]

where \( p_i \) means photometric parameters of galaxies, \( n \) is the number of parameters, \( c_0 \) the zero-point for reaching a minimal \( \chi^2 \), and \( \sigma_i \) photometric error for \( p_i \). We choose 9 parameters: five-band magnitudes \( u', g', r', i', z' \) and four colors \( u' - g', g' - r', r' - i', i' - z' \). According to the \( \chi^2 \)-distribution versus redshift, the most probable solution is calculated as the best estimate (\( z^{\text{fitting}} \)) for a given SED. The four color items seem to be the re-count of \( u', g', r', i', z' \) magnitudes. However, they represent the cross-items of the SDSS magnitude system, and thus can be selected as independent quantities in the \( \chi^2 \) minimization algorithm.

We select a sample of template SEDs from sample \( \mathcal{S} \) by following criteria that 1) photometric errors are less than 0.03 mag in \( r' \) band, and 2) the redshift confidence (variable \( z\text{Conf} \) in the SDSS database) must be greater than 0.7, and 3) the galaxies are evenly distributed in a range of \( z < 0.5 \). However, we have a small number of galaxies with \( z > 0.5 \), thus they are all included. We select 18,144 galaxies as templates according to the conditions. The result instead of using the r.m.s. estimate, unless specially mentioned. The biweight estimation, \( \sigma \), gives a more intrinsic dispersion, or core dispersion, while r.m.s. tends to be affected by the tail distribution.
of template fitting is shown in the left panels of Figure 5 and in Table 1 (method A). We use the same grouping strategy as we take in deriving CMR matrices (see Sect. 3.1). For each group, the detailed fitting configurations, such as typical photometric error, are adjusted carefully, because each group of magnitude has its own typical photometric precision. From the result, we notice that some low-redshift galaxies are misestimate to be high-redshift ones. We calculate that the fraction of galaxies with $|z_{\text{fitting}} - z_{\text{spec}}| < 0.05$ is about 90%, which implies that the template fitting method is effective except for a small fraction of scattering low-redshift galaxies.

### 3.3. Final PR Estimation

We have introduced two independent methods to estimate PRs of galaxies in the above two subsections. We find that the PR uncertainty can be statistically reduced by averaging the two PR estimates. And the shortcoming of SED-fitting method, the inefficiency of some low-redshift galaxies, can also be reduced by the combination. With the mean PR, $z_{\text{mean}}$, a further gain of PR precision can be obtained from a quadratic correction of photometric properties, which has been proved to be helpful to improve PR estimation (Connolly et al. 1995). We have equation:

$$\begin{align*}
z_{\text{PR}} &= a_0 z_{\text{mean}} + b_0 + A + B, \\
A &= \sum_{i=1}^{n} a_i p_i, \\
B &= a'_0 z_{\text{mean}}^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} p_i p_j, \\
b_0 &= \text{const},
\end{align*}$$

where $p_i$ and $n$ are the same as in Eq. (1), $A$ and $B$ are the first and second order terms respectively. As long as we solve the equation with the known SRs, the coefficients can be applied to other SDSS galaxies.

In brief, we need three steps to determine PRs of galaxies. First, we use the template fitting and CMR matrices to derive $z_{\text{fitting}}$ and $z_{\text{CMR}}$. Second, PR estimation can be improved by getting $z_{\text{mean}}$. Finally, we calculate $z_{\text{PR}}$ according to the solution of Eq. (2). However, if we simply follow these steps, we will get no obvious enhancement of PR estimation, because the relation between redshift and photometric properties is intrinsically non-linear and very complicated. Several detailed rules are made in order to achieve a better performance. We have divided sample $S$ into seven subsamples according to $r'$-band magnitudes of galaxies.
in deriving the CMR matrices (see Sect. 3.1 and Table 1). After calculating the $z_{\text{fitting}}$ and $z_{\text{CMR}}$ for each galaxy, we apply following Rule 1 for averaging $z_{\text{fitting}}$ and $z_{\text{CMR}}$.

**Rule 1**: For all galaxies, $z_{\text{mean}}$ equals the average of $z_{\text{CMR}}$ and $z_{\text{fitting}}$. But for the most scattering points (MSPs) characterized by $|z_{\text{fitting}} - z_{\text{CMR}}| > 0.05$ in sub-samples of R1 to R5, $z_{\text{mean}}$ must be replaced by $z_{\text{CMR}}$, because we find that $z_{\text{CMR}}$ converges statistically than other results for MSPs.

The results after applying Rule 1 are plotted in the middle panels of Figure 5, and listed in Table 1 (method B). It is obvious that the shortcoming of template fitting is alleviated significantly. Next, we select a set of subsamples to solve Eq. (2). To guarantee the stability and reliability of solution of Eq. (2), we use as many galaxies as possible to solve the equation, but not more than 20,000 galaxies for each solution. For R1, R2, R6 and R7, we just solve the equation for each group respectively. But for the left groups, we need the more detailed Rule 2 and 3:

**Rule 2**: For R3 and R4, the coefficients are solved for non-MSPs and MSPs respectively.

**Rule 3**: Rule for R5 is a little bit complex. The coefficients are derived for these four cases: a) non-MPS with $z < 0.3$; b) non-MPS with $z > 0.3$; c) MPS with $z < 0.3$; and d) MPS with $z > 0.3$.

Results after quadratic correction are shown in the right panels of Figure 5. It is clear that $z_{\text{PR}}$ is the best performance, which also can be easily found out from Table 1. The best result of PR is that $\sigma_{\text{PR}}$ is 0.0219 with a correlation factor between $z_{\text{spec}}$ and $z_{\text{PR}}$ of 0.959. Figure 6 shows a comparison of input SR distribution and the output PR distribution. Although there exists a little discrepancy at $z \sim 0.1$ and $z \sim 0.35$ the result is remarkably good by considering the typical PR uncertainty and the correlation factor. For further evaluating our method, we calculate the mean PR uncertainties indexed by $z_{\text{PR}}$ in different redshift intervals (see Table 2).

4. DISCUSSION

The goal of PR technique is to predict the redshifts of galaxies without spectroscopic observation. Thus it is necessary to check whether our method is practical, especially for high-redshift galaxies and faint galaxies. We apply the method presented above to the deep photometric data, sample $\mathcal{P}$ (see Sect. 2). The PR distributions of galaxies are shown in Figure 7 which suggests that the PR distribution for galaxies brighter than $r' = 20$ is well in agreement with the theoretical prediction (Baugh & Efstathiou 1993). For a limiting
magnitude of \( r' = 21 \), the distribution appears to have a little distortion, but still acceptable. When the limiting magnitude is set to be \( r' = 21.25 \), obvious distortion appears at \( z > 0.5 \). The method fails at faint magnitude partly due to the lack of templates with known SRs or possibly due to the bias of template discussed in Sect. 2.

We have presented a new PR technique, which leads to a good result for galaxies brighter than \( r' \sim 21 \) and turns out to be stable in the application of both spectroscopic sample and photometric sample. Majority of galaxies (near 90%) in sample \( S \) are brighter than \( r' \approx 18 \). Therefore, the typical uncertainty we quote for the final PR is mainly contributed by these bright galaxies with very good photometry (the typical \( \delta r' < 0.02 \)). During the whole process, we do not exclude any types of galaxies. From the test of unbiased sample (i.e., \( r' < 18 \)), it shows that the method presented here is a good tool to estimate PR for all types of galaxies. As for MSPs, we treat them as galaxies with special behaviours no matter what type they are. We investigate MSPs in sample \( S \) and find that their photometry is normal (\( \delta r' \sim 0.01 \)), and that most of them are spiral galaxies morphologically\(^2\).

The three steps, which are taken to determine PR, are all important. \( z_{\text{fitting}} \) and \( z_{\text{CMR}} \) estimates are complementary in redshift domain. Statistically, the latter gives a slightly better result than the former dose for brighter \( r' < 18 \) galaxies, and it alleviates the shortcoming of the former. Moreover, \( z_{\text{fitting}} \) compensates the incompleteness of \( z_{\text{CMR}} \). Of course, this kind of incompleteness can be reduced by enlarging the spectroscopic sample for CMR. Their “average”, \( z_{\text{mean}} \), is an initial estimation for further quadratic correction. If we simply use quadratic fitting to the photometric properties, as did in Connolly et al. (1995), we cannot get the results as good as we present above.

Another important step is that we divide the dataset into different groups by magnitude. The photometric error strongly affects PR precision, which can be found in Table 1 and is also demonstrated by simulation (Xia et al. 2002). The strong correlation implies that as long as we achieve the best photometry, PR can be determined as precisely as \( \sigma_{\text{PR}} \sim 0.015 \), just as the bright galaxies in R1 sub-sample with a typical photometric error is \( \delta r' < 0.01 \). Consequently, it is reasonable and necessary to individually deal with the magnitude-dividing groups with different photometric properties.

We would like to highlight that the relation between redshift and photometric properties is strongly non-linear, making the individual methods successful in some ranges, but poorer in others. Therefore the complementary combination of multiple methods improves the PR precision significantly. The present method of PR estimation is practical. However, due to

\(^2\)We use the SDSS DR4 Image List Tool for inspection, see http://cas.sdss.org/astro/en/tools/chart/list.asp
the lack of faint templates and large photometric error of faint SDSS galaxies, this method is effective only for the galaxies brighter than \( r' \sim 21 \).

5. SUMMARY

We present and evaluate a practical method to estimate the PRs of galaxies with Sloan five-band photometry. A comprehensive application of three independent methods leads to a good PR estimate of the SDSS galaxies. First, by taking observed SEDs as template, we perform \( \chi^2 \)-minimization fitting to derive the redshifts of galaxies. Second, as the compensation for the method of template fitting, the digitalized CMR relations are introduced for PR estimate. Then we find that the precision of PR estimation can be improved by “averaging” the PRs derived from CMRs and template fitting. Furthermore, a quadratic correction based on photometric properties of the SDSS galaxies is applied to the initial “average” redshifts, which leads to an enhancement of our final PR estimation. During these processes, some specific rules are configured in order to achieve the best performance of PR estimation. Finally we obtain a good result with the typical error of \( \sigma_{PR} \sim 0.0219 \). By combining the advantages of several methods, our algorithm improves PR estimation of galaxies significantly, and stands the test of the huge spectroscopic sample and the photometric sample. The application to photometric data suggests that with the available Sloan spectroscopic sample our method can be applied to the SDSS galaxies down to \( r' \sim 21 \) mag.

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Table 1: PR estimation for magnitude-dividing sub-samples.

<table>
<thead>
<tr>
<th>Magnitude range</th>
<th>( N_{\text{gal}} )</th>
<th>( \delta r^a )</th>
<th>( \delta z^b )</th>
<th>( \sigma^c )</th>
<th>r.m.s. ( ^c )</th>
<th>( r^d )</th>
<th>( M^e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1 ( r' &lt; 16.0 )</td>
<td>43,459</td>
<td>0.003</td>
<td>-0.006</td>
<td>0.0218</td>
<td>0.0333</td>
<td>0.492</td>
<td>A</td>
</tr>
<tr>
<td>R2 ( 16.0 &lt; r' &lt; 17.0 )</td>
<td>120,373</td>
<td>0.005</td>
<td>0.007</td>
<td>0.0231</td>
<td>0.0317</td>
<td>0.742</td>
<td>A</td>
</tr>
<tr>
<td>R3 ( 17.0 &lt; r' &lt; 18.0 )</td>
<td>235,456</td>
<td>0.008</td>
<td>0.013</td>
<td>0.0275</td>
<td>0.0364</td>
<td>0.839</td>
<td>A</td>
</tr>
<tr>
<td>R4 ( 18.0 &lt; r' &lt; 19.0 )</td>
<td>42,921</td>
<td>0.015</td>
<td>0.006</td>
<td>0.0342</td>
<td>0.0483</td>
<td>0.878</td>
<td>A</td>
</tr>
<tr>
<td>R5 ( 19.0 &lt; r' &lt; 20.0 )</td>
<td>15,462</td>
<td>0.028</td>
<td>0.007</td>
<td>0.0341</td>
<td>0.0559</td>
<td>0.813</td>
<td>A</td>
</tr>
<tr>
<td>R6 ( 20.0 &lt; r' &lt; 21.0 )</td>
<td>1,952</td>
<td>0.059</td>
<td>0.022</td>
<td>0.0927</td>
<td>0.1482</td>
<td>0.610</td>
<td>A</td>
</tr>
<tr>
<td>R7 ( 21.0 &lt; r' &lt; 23.0 )</td>
<td>1,429</td>
<td>0.156</td>
<td>0.041</td>
<td>0.2129</td>
<td>0.2355</td>
<td>0.295</td>
<td>A</td>
</tr>
<tr>
<td>R8 All galaxies</td>
<td>461,170</td>
<td>—</td>
<td>0.009</td>
<td>0.0274</td>
<td>0.0412</td>
<td>0.929</td>
<td>A</td>
</tr>
</tbody>
</table>

\( a \)Typical photometric error of each magnitude interval.
\( b \)Bias between PRs and SRs.
\( c \)Two kinds of estimation of PR uncertainty.
\( d \)Linear correlation factor between PRs and SRs.
\( e \)Different methods of PR: A represents the results of template fitting, \( z_{\text{fitting}} \) (see Sect. 3.2), B \( z_{\text{mean}} \) for the “average” of \( z_{\text{fitting}} \) and \( z_{\text{CMR}} \) (see Sect. 3.1), C for the result after a linear correction, \( z_{PR} \) (see Sect. 3.3).
Table 2: Variation of PR uncertainties with redshift.

<table>
<thead>
<tr>
<th>Redshift Range (z_{PR})</th>
<th>δz</th>
<th>σ_{PR}</th>
<th>r.m.s._{PR}</th>
<th>N_{gal}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0 &lt; z &lt; 0.1</td>
<td>&lt; 0.001</td>
<td>0.021</td>
<td>0.024</td>
<td>202126</td>
</tr>
<tr>
<td>0.1 &lt; z &lt; 0.2</td>
<td>−0.001</td>
<td>0.021</td>
<td>0.025</td>
<td>175298</td>
</tr>
<tr>
<td>0.2 &lt; z &lt; 0.3</td>
<td>0.000</td>
<td>0.023</td>
<td>0.040</td>
<td>32276</td>
</tr>
<tr>
<td>0.3 &lt; z &lt; 0.4</td>
<td>−0.002</td>
<td>0.027</td>
<td>0.040</td>
<td>33664</td>
</tr>
<tr>
<td>0.4 &lt; z &lt; 0.5</td>
<td>−0.004</td>
<td>0.024</td>
<td>0.051</td>
<td>14128</td>
</tr>
<tr>
<td>0.5 &lt; z &lt; 0.6</td>
<td>0.001</td>
<td>0.037</td>
<td>0.102</td>
<td>2320</td>
</tr>
<tr>
<td>0.6 &lt; z &lt; 0.7</td>
<td>−0.002</td>
<td>0.114</td>
<td>0.129</td>
<td>974</td>
</tr>
</tbody>
</table>

NOTE: Some explanation of the columns can be found in Table 1.
Fig. 1.— The $g'\!-\!r'\!-\!i'$ diagrams for 62,512 randomly selected galaxies brighter than $r' < 19$ in sample $S$. Panel (a) to (d) are corresponding to galaxies with $r' < 16$, $16 < r' < 17$, $17 < r' < 18$ and $18 < r' < 19$, respectively. Redshifts of galaxies are shown as gray points, brighter for lower redshifts and blacker for higher redshifts.
Fig. 2.— CMR matrices (see Sect. 3.1) are shown in the form of images. Different redshifts are indicated by gray, brighter for lower redshift while darker for higher redshift. Each row of panels (R1 to R7) corresponds to different magnitude ranges that can be found in Table 1.
Fig. 3.— PR estimations ($z_\text{I}$ and $z_\text{II}$) from CMR matrices are compared with the $z_{\text{spec}}$. We convert the dot distribution to a logarithmic density map. This process is applied to other similar figures, such as Figures 4 and 5. The contour levels are 95%, 75%, 55% and 35% of the maximum.
Fig. 4.— The combination of $z_1$ and $z_2$, $z_{CMR}$ is compared with $z_{spec}$. The contour levels are the same in Figure 3.
Fig. 5.— (to be continued) From top to bottom, each row shows PR estimations of magnitude-dividing samples. The bottom row (i.e., R8) shows the result from the whole spectroscopic sample. The left, middle and right panels present the result of $z_{\text{fitting}}$, $z_{\text{mean}}$, and $z_{\text{PR}}$, respectively. The contour levels are the same in Figure 3.
Fig. 5. — continued.
Fig. 6.— For sample $S$, we compare the input spectroscopic redshift distribution (dotted line) and the output distribution of PR estimation (solid line).
Fig. 7.— Histogram of PR estimation of photometric sample: panel (a) for 49,130 galaxies with $r' < 19$, (b) for 138,242 galaxies with $r' < 20$, (c) for 382,621 galaxies with $r' < 21$ and (d) for 486,818 galaxies with $r' < 21.25$. The median redshift of the distributions are 0.17, 0.24, 0.35, 0.41 for (a) to (d) respectively. The solid curves indicate the theoretical prediction of galaxy distribution in redshift space with corresponding magnitude limits.